Problem 6.15. Calculate the mean energy of an ideal gas of $N = 2$ identical bosons in equilibrium with a heat bath at temperature T, assuming that each particle can be in one of three states with energies, 0, Δ, and 2Δ. Is it possible to reduce this problem to a one body problem as we did for a noninteracting classical system?

Solution. The number of bosons in each microstate is given in Table 1.

microstate s	0	Δ	2Δ	E_s
1	2	0	0	0
2	0	2	0	2Δ
3	0	0	2	4Δ
4	1	1	0	Δ
5	0	1	1	3Δ
6	1	0	1	2Δ

Table 1: The microstates and their energies of a system of two bosons, each of which can have 3 possible energies. The numbers in columns two to four give the occupation numbers of the single particle states.

From Table 1 we see that

$$Z = 1 + e^{-\beta\Delta} + 2e^{-2\beta\Delta} + e^{-3\beta\Delta} + e^{-4\beta\Delta} \tag{1}$$

and

$$E = \frac{\Delta}{Z}[e^{-\beta\Delta} + 4e^{-2\beta\Delta} + 3e^{-3\beta\Delta} + 4e^{-4\beta\Delta}. \tag{2}$$

Z cannot be reduced to a simpler form.