

Problem 3.63. In the World Series in baseball and in the playoffs in the National Basketball Association and the National Hockey Association, the winner is determined by a best of seven series. That is, the first team that wins four games wins the series and is the champion. Do a simple statistical calculation assuming that the two teams are evenly matched and the winner of any game might as well be determined by a coin flip and show that a seven game series should occur 31.25% of the time. What is the probability that the series lasts n games?

Solution. In order for the series to end after four games, one team must win the first four games in a row:

$$(1/2)(1/2)(1/2)(1/2) + (1/2)(1/2)(1/2)(1/2) = 2/16 = 1/8 = 0.125. \quad (1)$$

For the series to end in 5 games, one team must win exactly 3 out of the first 4 games (in any order) and then win the fifth game:

$$C(4, 3)(1/2)(1/2)(1/2)(1/2)(1/2) + C(4, 3)(1/2)(1/2)(1/2)(1/2)(1/2) = 8/32 = 1/4 = 0.25. \quad (2)$$

In order for the series to end in 6 games, one team must win exactly 3 out of the first 5 games (in any order) and then win the sixth game:

$$\begin{aligned} C(5, 3)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2) + C(5, 3)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2) &= 20/64 \\ &= 5/16 = 0.3125. \end{aligned} \quad (3)$$

In order for the series to end in 7 games, one team must win exactly 3 out of the first 6 games (in any order) and then win the seventh game:

$$\begin{aligned} C(6, 3)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2) + C(6, 3)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2)(1/2) \\ = 40/128 = 5/16 = 0.3125. \end{aligned} \quad (4)$$