Chapter 5: 10,15 - deadline Friday (11/17)

Problem 3

Obtain the generating functions F_2 from F_1 through the application of the Legendre transformation:

$$F_2(q, P, t) := \sum_i P_i Q_i + F_1(q, Q, t)$$

and show that

$$p_i = \frac{\partial F_2}{\partial q_i}, \quad Q_i = \frac{\partial F_2}{\partial P_i}, \quad K = H + \frac{\partial F_2}{\partial t}.$$

Problem 4

The transformation equations between two sets of coordinates are

$$Q = \frac{q^2 + p^2}{2}, \qquad P = -\tan^{-1}\frac{q}{p}. \tag{1}$$

In the class we have derived the generating function F_1 for the above transformation

$$F_1 = F_1(q, Q) = Q \tan^{-1}\frac{q}{(2Q - q^2)^{1/2}} + \frac{1}{2}q(2Q - q^2)^{1/2}.$$

(a) Use the generating function F_1 and generating function derivatives (see 9.14a-9.14c equations in the textbook) to derive the canonical transformation (1).

(b) If the old Hamiltonian is

$$H = \frac{1}{2}(q^2 + p^2),$$

find the new Hamiltonian $K = K(Q, P, t)$.

(c) Compare the canonical equations of motion in the old coordinates and the new coordinates. Make a connection to the simple harmonic oscillator, i.e. identify the mass m and the spring constant k.

(d) Solve the equations of motion in both coordinate systems and provide an interpretation to your solution. Keep in mind that the position and the momentum is easily switched by $F_1 = \sum_i q_i Q_i$.