

Example 4.1

A uniform plank of mass M is leaning against a smooth wall and makes an angle α with the smooth floor, as shown in Figure 4.3. The lower end of the plank is connected to the base of the wall with a massless inextensible string. What is the tension in the string?

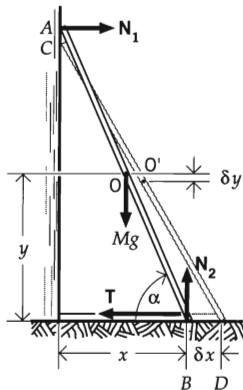


Figure 4.3 A plank against a smooth wall.

Solution

This example shows a scleronomic system with workless constraints. The external constraint forces are the wall and floor reactions \mathbf{N}_1 and \mathbf{N}_2 . They do no work.

We take the length of the plank as $2b$. The midpoint O of the plank is at a distance y above the floor, and the lower end of the plank is at a distance x from the wall, where

$$x = 2b \cos \alpha, \quad y = b \sin \alpha \quad (4.15)$$

The forces acting on the plank are:

\mathbf{T} = tension in the string;

$M\mathbf{g}$ = weight of the plank;

$\mathbf{N}_1, \mathbf{N}_2$ = normal reactions of the wall and the floor, respectively.

We now imagine a horizontal displacement δx of the lower end of the plank from B to D . The change in the level of O is δy . From Eq. 4.15, we see that

$$|\delta y| = b \cos \alpha \delta \alpha, \quad |\delta x| = 2b \sin \alpha \delta \alpha \quad (4.16)$$

Now the constraint forces \mathbf{N}_1 and \mathbf{N}_2 do no work, because the wall and the floor are smooth (frictionless). Then the principle of virtual work gives

$$-T|\delta x| + Mg|\delta y| = 0 \quad (4.17)$$

where the sign convention is that the tension T and δx are in opposite directions, so it does negative work. Conversely, while δy and Mg are in the same direction, Mg does positive work.

Substituting Eq. 4.16 into Eq. 4.17 and solving for T , we find

$$T = \frac{1}{2}Mg \cot \alpha$$

Example 4.2

A ring of mass m slides on a smooth vertical rod (Fig. 4.4). Attached to the ring is a light inextensible string passing over a smooth peg distance a from the rod. At the other end of the string is a mass M ($M > m$). The ring is released from rest at the same level as the peg. If h is the maximum distance the ring will fall, determine h in terms of M , a , and m .

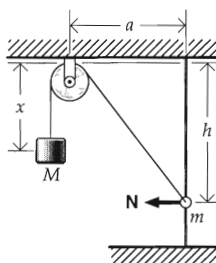


Figure 4.4 A ring slides on a vertical rod.

Solution

If b is the length of the string, then clearly

$$x + (h^2 + a^2)^{1/2} = b \quad (4.18)$$

Now imagine a vertical displacement δh of the ring along the rod. The change in x is connected to h by the following equation

$$\delta x + (h^2 + a^2)^{-1/2} h \delta h = 0$$

or

$$|\delta x| = (h^2 + a^2)^{-1/2} h |\delta h| \quad (4.19)$$

The constraints over the rod and peg do no work, so the principle of virtual work gives

$$Mg|\delta x| + (-mg|\delta h|) = 0 \quad (4.20)$$

where the sign convention is that x and Mg are in the same direction, so Mg does positive work; while h and mg are in opposite directions, mg does negative work.

Substituting Eq. 4.19 into Eq. 4.20, we obtain

$$h = \frac{ma}{(M^2 - m^2)^{1/2}}$$