A Generalized Asset Exchange Model with Economic Growth and Wealth Distribution

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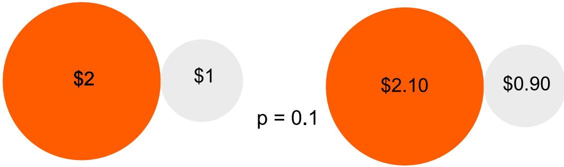
How does the distribution of wealth arise from microeconomic interactions?



- N agents with wealth w_i and $\sum_{i=1}^{N} w_i = N$.
- Chose two agents at random and winner with probability $1/2$.
- Transfer fraction p of the poorer agent's wealth from the loser to the winner.
- What is the resulting wealth distribution?
- Counterintuitive result: One agent continues to gain almost all the wealth and all others have almost none.

How does economic growth and its distribution affect the distribution of wealth?

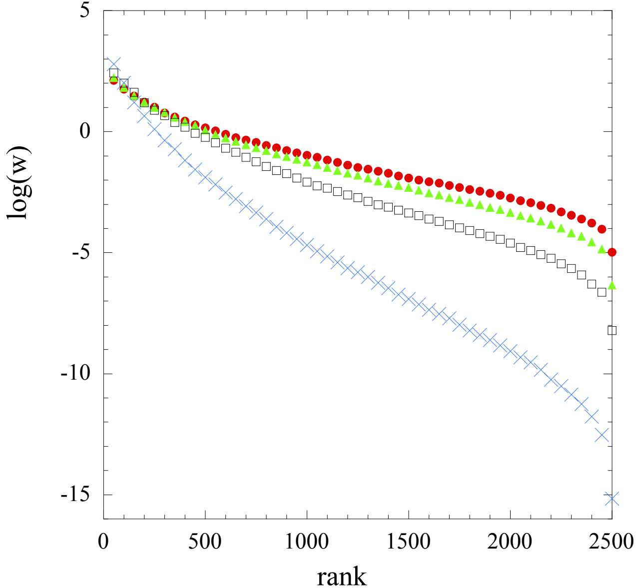
- After N exchanges a fraction μ of the total wealth is added to the system (geometrical growth).
- Additional wealth given to agent i:

$$\Delta w_i(t) = \mu W(t) \frac{w_i^{\lambda}(t)}{\sum_{i=1}^{N} w_i^{\lambda}(t)}$$

$$W(t) = \sum_{i=1}^{N} w_i(t)$$

  $\lambda \geq 0$ is a distribution parameter.
- After distribution rescale w_i so that $\sum_i w_i = N$.

A rising tide lifts all boats for $\lambda < 1$



Steady state rescaled wealth distribution versus rank for $\lambda = 0.2$ (●), 0.4 (▲), 0.6 (□), and 0.8 ($\times$). Wealth distribution less equal as $\lambda \to 1^-$.

Qualitatively different behavior for $\lambda < 1$ and $\lambda \geq 1$

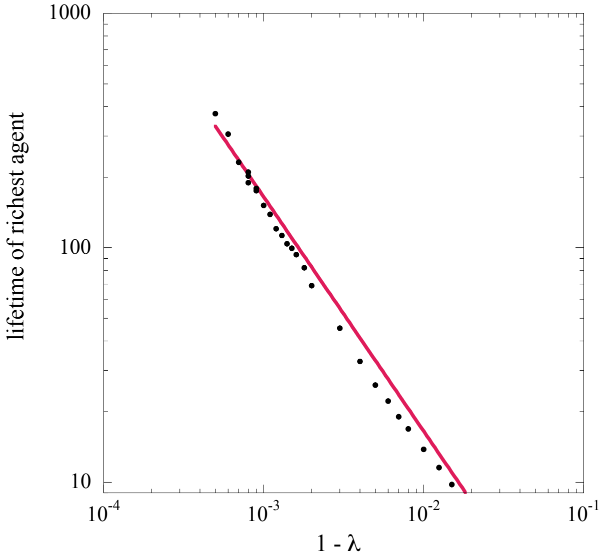
$\lambda < 1$

- No wealth condensation.
- Rescaled wealth distribution reaches a steady state.
- Greater wealth equality as $\lambda \to 0$.
- Economic mobility: richer agents become poorer and vice versa.
- System in thermal equilibrium.

$\lambda \geq 1$

- Wealth condensation as in model without growth, no mobility, no steady state.

Critical slowing down: Lifetime of richest agent



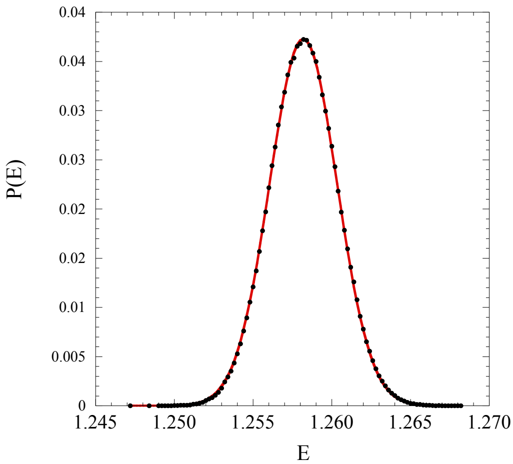
$$\tau \sim (1-\lambda)^{-1}.$$

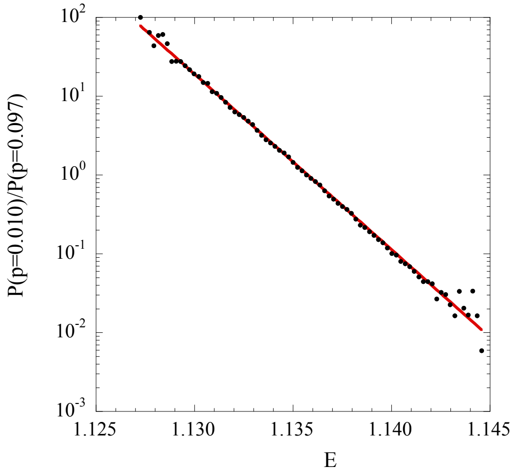
Existence of critical slowing down limits simulations near $\lambda = 1^{-}$.

Equilibrium – not just steady state

$$E = \frac{1}{N}\sum_i w_i^2$$ Total energy

$$P(E) = \Omega(E)e^{-\beta E}$$ Probability density





$N = 5000$, $\lambda = 0.5$. (a) $P(E)$ for $p = 0.1$. (b) $P(p = 0.1)/P(0.097)$.
Equilibrium more difficult to verify as $\lambda \to 1$.

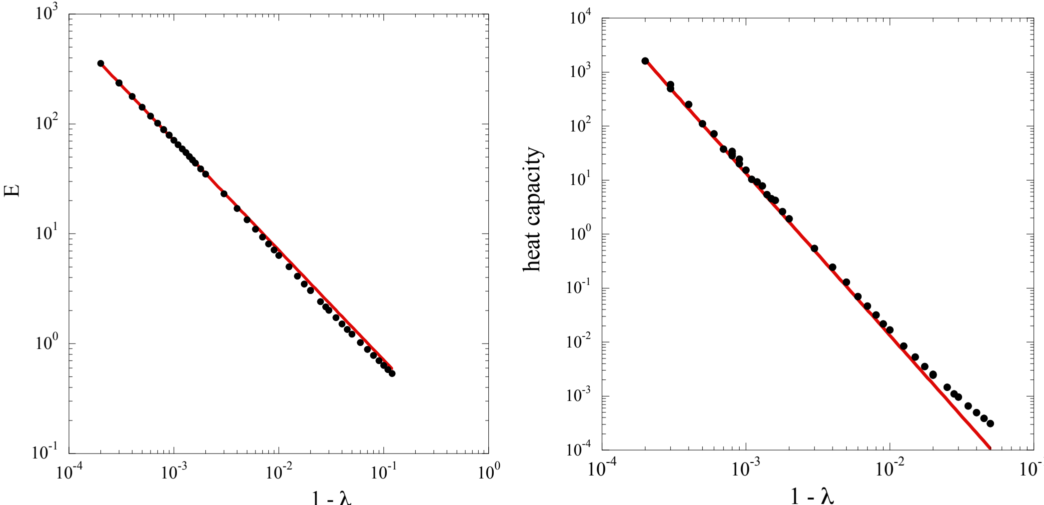
Phase transition at $\lambda = 1$ is critical point

- Phase transition is continuous with critical exponents that characterize the transition.
- Susceptibility χ: Fluctuations of order parameter.
- Order parameter: Fraction of wealth held by all agents but the richest.

Simulations for $N = 5000$, $\mu = 0.1$. Exponents independent of μ.

Critical exponents for constant N

- $\beta_N = 0$, $\gamma_N = 2$, $\alpha_N = 3$, $\alpha_N + 2\beta_N + \gamma_N \neq 2$.
- Total energy diverges as $\lambda \to 1^-$.



Fits assume $E \sim (1-\lambda)^{-1}$ and $\alpha_N = 3$.

Mean-field theory

- Mean-field theory based on exchange of wealth between agent chosen at random and agent whose wealth equals mean wealth of the remaining agents.
- Mean-field theory self-consistent if

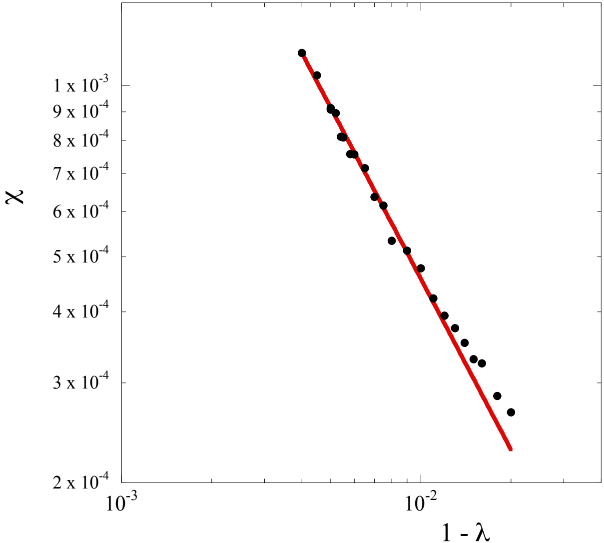
$$\text{Ginzburg parameter } G = N\mu(1-\lambda) \gg 1$$

  and held constant as $\lambda \to 1$.
- Predictions: $\beta = 0$, $\gamma = 1$, $\alpha = 1$, $\alpha + 2\beta + \gamma = 2$.
- Total energy approaches a constant as $\lambda \to 1^-$.
- Time scale for critical slowing down

$$\tau \sim (1-\lambda)^{-1}.$$

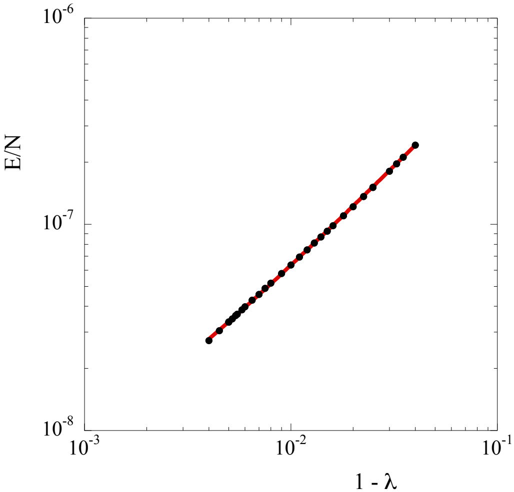
Constant Ginzburg parameter

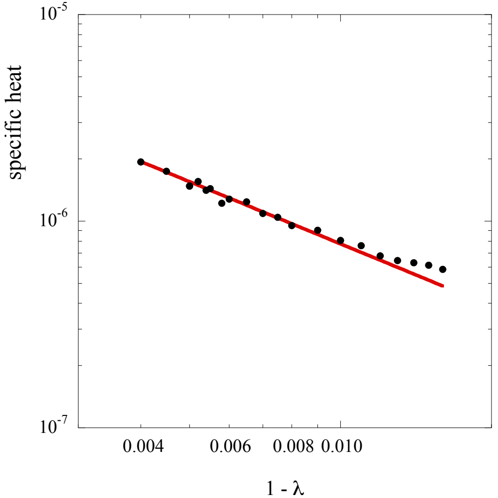
$500 \leq N \leq 20000$ and $0.996 \leq \lambda \leq 0.800$, $G = 10$.



Fit assumes $\chi \sim (1-\lambda)^{-1}$.

Energy and specific heat





- Fits assume $E/N \sim \text{const} + (1 - \lambda)$.
- $C \sim (1 - \lambda)^{-1}$.
- $E \propto N$ only if G held fixed.

Discussion

- Numerical results for exponents consistent with mean-field theory predictions.
- No wealth distribution leads to wealth condensation. All benefit if distribution favors wealthy and $\lambda < 1$.
- System becomes more mean-field as $N \to \infty$.
- As globalization increases, do mean-field models of the global economy become more relevant?
- Wealth of the richest agent grows exponentially if the system is "quenched" from $\lambda < 1$ to $\lambda > 1$. Further evidence that the transition can be interpreted as a spinodal in the mean-field limit.

Discussion and future work

- Because the model is in equilibrium for $\lambda < 1$, are there aspects of the economy that are treatable by equilibrium statistical mechanics?
- The mean-field theory yields a stochastic differential equation with both additive and multiplicative noise.
- If only additive noise is retained, critical exponents can be predicted and wealth distribution is Gaussian.
- If both types of noise are included, numerical solutions show that wealth distribution is log-normal, consistent with the agent-based simulations. Can we obtain an analytical solution?
- Make contact with economic data. Preliminary work suggests $\lambda \approx 0.8$.
- Generalize the model so that growth is not imposed externally.

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