Simple models of earthquake faults: Examples of driven dissipative systems

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Motivation: Gutenberg-Richter scaling

Total number of events with moment greater than or equal to moment M (energy)

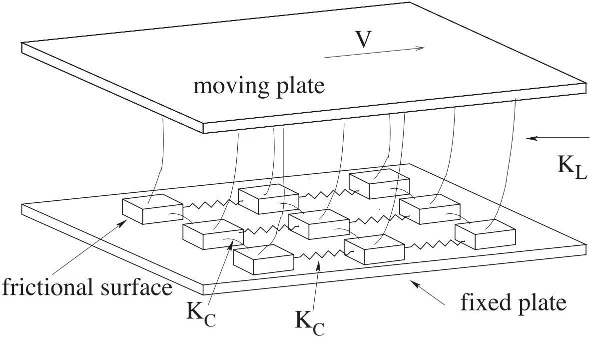
$$N_M \propto M^{-\beta} \qquad (\beta \approx 2/3)$$

Is Gutenberg-Richter scaling related to critical phenomena and self-organized criticality?

Will discuss three models in order of decreasing complexity that yield power law scaling:

- Burridge-Knopoff, blocks and springs; need to solve Newton's equations.
- Rundle-Jackson-Brown, stress and energy, cellular automaton.
- Olami-Feder-Christensen (OFC), stress, cellular automaton.

Burridge-Knopoff slider-block model (1967)



Blocks connected by springs and coupled by springs to loader plate and pulled over a rough surface described by a velocity-weakening friction law.
Need to solve Newton's equations numerically.
Carlson and Langer (1989); Xia, Gould, Klein, and Rundle (2005).

Rundle, Jackson, Brown (RJB) Cellular Automaton Model

Cellular automaton version of Burridge-Knopoff model. Ignore inertia effects. Each site has stress σ_i and energy ϵ_i. Rundle and Jackson (1977), Rundle and Brown (1991).

The evolution of the stress is same as Olami, Feder, and Christensen model (1992) so will emphasize the latter.

Olami, Feder, and Christensen (OFC) model

Stress σ_i on each site i with failure threshold σ_F and mean residual stress σ_R. Dissipation coefficient α. Initially distribute stress at random.

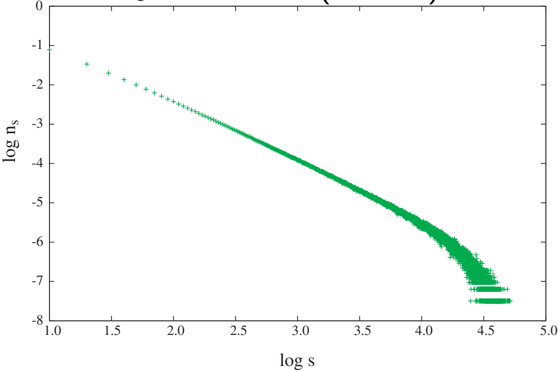
1. If $\sigma_i \geq \sigma_F$, reduce σ_i to $\sigma_R + \eta$ and distribute stress $(1-\alpha)(\sigma_i - \sigma_R - \eta)$ to neighbors; η represents noise.
2. Check neighbor sites and go to step 1.
3. Continue until $\sigma_i < \sigma_F$ for all i. Number of sites that fail constitutes an earthquake of size s.
4. Reload system: Find site with maximum stress and bring it to failure by adding $\sigma_F - \sigma_{\max}$ to all sites.

What have we learned?

• n_s, number of events of size s, does not scale if stress distributed only to nearest neighbors ($R = 1$).

• Give stress equally to all sites within distance $R \gg 1$. Long-range mimics elastic force.

$$n_s \sim s^{-3/2} \quad (s \gg 1)$$



$R = 30$, $\alpha = 0.01$, $\sigma_F = 1.0$, $\sigma_R = 0.25 \pm 0.25$, $L = 256$,

More results for $R \gg 1$ (near-mean field)

• Form of Langevin equation same as for Ising model. $\Longrightarrow$ OFC model in equilibrium for $R \gg 1$.

$$\ln P(E) \propto E, \qquad E = \frac{1}{2}\sum_{i=1}^{N} \sigma_i^2 \quad \text{(OFC model)}.$$

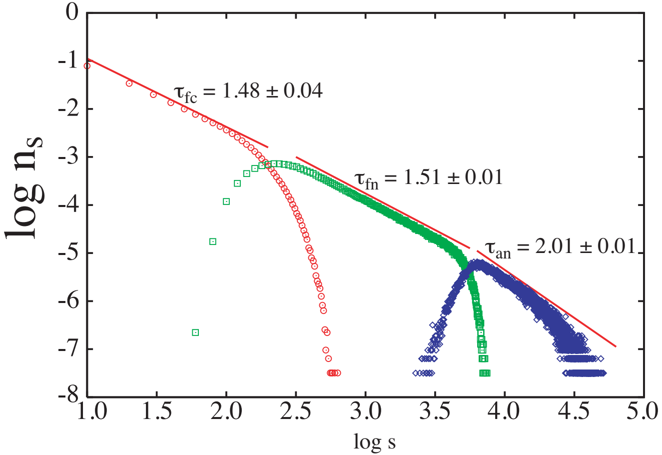
• Scaling of events similar to scaling of clusters in mean-field Ising model near spinodal.

$$n_s \sim s^{-\tau} e^{-\Delta h\, s^{\sigma}} \qquad (\tau = 5/2,\ \sigma = 1)$$

($\tau = 7/3$ for Ising mean-field critical point.)
Ising clusters generated by tossing random bonds between parallel spins. Earthquakes grow from a single site.

$$s\, n_s(\text{random bond}) = n_s(\mathrm{seed})$$

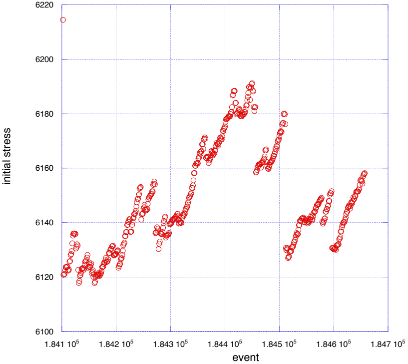
Spread of n_s $s \gg 1$ not due to poor statistics

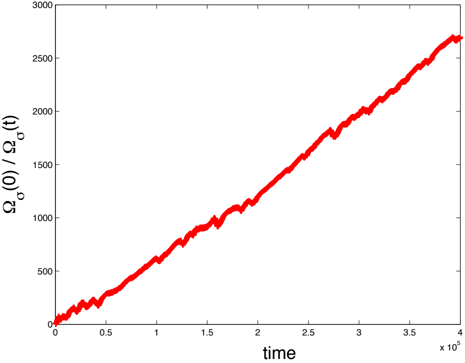


$$\tau = \begin{cases} 3/2 & \text{clusters} & \sigma_{\min} \in [0.99,\ 1.0) \\ 3/2 & \text{failed nucleation} & \sigma_{\min} \in [0.90,\ 0.99) \\ 2 & \text{arrested nucleation} & \sigma_{\min} \in [0.0,\ 0.90) \end{cases}$$

Time between given type of event $\propto e^{-t/\tau}$.

Punctuated equilibrium





$$\overline{\sigma}_i(t) = \frac{1}{t}\int_0^t \sigma_i(t')dt' \qquad <\sigma(t)> = \frac{1}{N}\sum_{i=1}^{N}\overline{\sigma}_i(t).$$

$$\Omega(t) = \frac{1}{N}\sum_{i=1}^{N}\left[\bar{\sigma}_i(t) - <\sigma(t)>\right]^2$$ (Thirumalai-Mountain)

If system is effectively ergodic, $\Omega(t) \propto 1/t$.

Gutenberg-Richter scaling [Serino et al. (2011)]

• OFC model describes *individual faults* rather than fault systems. The latter satisfy Gutenberg-Richter scaling, single faults may or may not.

• $s \propto M$ in mean-field limit.

$\beta = \tau - 1 = (3/2) - 1 = 1/2 \quad$ [empirical $\beta \approx 2/3$]

• Randomly eliminate fraction q of sites. Dead sites due to different levels of fracture or gouge. When stress is transferred to a dead site, it is dissipated.

• Model fault system as collection of noninteracting OFC models or "faults" with different values of q.

$$\tilde{n}_s \sim \int_0^1 dq\, D(q) n_s(q) \sim \int_0^1 dq\, \frac{1}{q^x} \left[\frac{1}{(1-q)} \frac{e^{-q^2 s}}{s^\tau} \right] \sim \frac{1}{s^{\tilde{\tau}}}.$$

$$\tilde{\tau} = 2 - x/2 \quad \implies \quad x = 2/3.$$

What about foreshocks, aftershocks, and quasiperiodic large events?

- In addition to three types of scaling events, there exist rare large “breakout” events that do not scale.
- Precursors and aftershocks exist in the OFC model for $R = 1$, but not for $R \gg 1$.
- Add asperity sites with larger σ_F. Find foreshocks prior to a main shock followed by aftershocks.
- Spatial and temporal patterns observed in natural seismicity are strongly influenced by the underlying physical properties and are not solely the result of a simple cascade mechanism [Kazemian et al. (2015)].

Burridge-Knopoff model

• Larger $\alpha_{\mathrm{BK}} \implies$ friction force decreases more rapidly with increasing velocity.

• $R = 1$ (no theory):

$$P(M) \sim M^{-2} \qquad (\alpha_{\mathrm{BK}} \gtrsim 1).$$

$M \propto \sum_i u_i$; $u_i =$ displacement of block i; sum over blocks in event. Larger events have characteristic size. No power law behavior for n_s.

• $R \gg 1$. One block connected to $2R$ neighbors with rescaled spring constant k/R.

$$n_s \sim \begin{cases} s^{-2} & (\alpha_{\mathrm{BK}} < 1) \qquad \text{no theory} \\ s^{-3/2} & (\alpha_{\mathrm{BK}} > 1) \qquad \text{same as OFC model} \end{cases}$$

Summary

- Olami-Feder-Christensen and Rundle-Jackson-Brown models effectively ergodic for $R \gg 1$.
- Gutenberg-Richter scaling associated with a pseudospinodal.
- Behavior of $R \gg 1$ Burridge-Knopoff model depends on how quickly friction force decreases with increasing velocity.
- Real earthquake faults can have different behavior depending on friction force, range of stress transfer, and presence of asperities.
- Does $R = 1$ OFC have a critical point at $\alpha = 0$?
- Do other driven dissipative systems become effective ergodic in mean-field limit?

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