Progress Report on Earthquake Project

Junchao Xia

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1 Improvements

I made two improvements to get results closer to Carlson and Langer's:

First a new criteria was used to judge if a block is stuck or not: At time $t = t_n$ a moving block is "stuck" only if

(a) The absolute value of its speed, V_n, relative to the substrate is less than an arbitrary value such as $V_0 = 0.01$;

(b) The direction of its relative velocity and the total force (including friction) at time $t = t_{n-1}$ are opposite. (We do so at $t = t_{n-1}$ because the magnitude of the friction force at $t = t_n$ depends on whether a block is stick or not.

(c) The "driving" force F_{drive} due to its neighbors and the loading plate is less than the maximum value of the static force, which is one in our choice of units.

Note that in (b) we have to use the information at time $t = t_{n-1}$. But there is no previous time step at $t = 0$. So at $t = 0$, I only use (a) and (c) to judge if a block is stuck.

A stuck block will accelerate when the driving force is larger than the maximum value of the static friction force, $F_{\text{drive}} > 1$.

After knowing a bolock is stuck or not, the dynamic or static friction force can be included in the total force as follows:
For a given block, I calculate F_{drive} and compare it to the maximum value of the static force. If $F_{\text{drive}} < 1$ and if the block was stuck, then we choose the static friction force, f_{fric}, so that it exactly cancels F_{drive} and the total force

is zero. (Recall that the static friction force is multiple valued if the velocity is zero. in other words, I treat the block as if it had zero velocity if it is stuck.) Otherwise, the maximum static force can't cancel F_{drive}, and we have to calculate the dynamic friction force f_{dyn} and add it to the driving force to get the total force on the block. (In this case, f_{dyn} is given by Eq. (5.1) of Carlson and Langer.

The following algorithm takes both the static and dynamic friction forces into account:

1. Initialize the system:
   (a) Give small random values (0.0 to 0.1) to all blocks in the system as initial displacements.
   (b) Set the relative velocities of all the blocks to zero;
   (c) Calculate the driving froce F_{drive}.
   (d) Use (a) and (c) to judge if a block is stuck or not;
   (e) If a block is stuck, then choose the static friction force f_{stat} such that the total force is zero; otherwise calculate the dynamic friction force f_{dyn};
   (f) Calculate the total force F_{tot} by adding driving force to the friction force.
2. Surpose that at $t = t_n$ we know the displacement $U(t_n)$, velocity $V(t_n)$, driving force $F_{\mathrm{drive}}(t_n)$ due to its neighbors and the loading plate, the dynamic friction force $f_{\mathrm{dyn}}(t_n)$ or static friction force $f_{\mathrm{stat}}(t_n)$, and hence the total force $F_{\mathrm{tot}}(t_n)$ on a given block. Then we use this information to get the corresponding values $U(t_{n+1})$, $V(t_{n+1})$,$F_{\mathrm{tot}}(t_{n+1})$ at time $t = t_{n+1}$ by using Shampine and Watts' fourth-order Runge-Kutta algorithm for Newton's equations of motion. Note that before we calculate $F_{\mathrm{tot}}(t_{n+1})$, $F_{\mathrm{drive}}(t_{n+1})$ is calculated first, then we use the method described above to judge a block is stuck or not and get $f_{\mathrm{dyn}}(t_{n+1})$ or $f_{\mathrm{stat}}(t_{n+1})$.
3. Use the velocity information to perform the analysis of the events (clusters);

The other change that I made was to recalculate the moment of an event. The moment is the sum of the *relative* displacements of the moving blocks involved in an event. Since in the equation of motion, the displacements are relative to the loading plate which is at rest, I have to calculate the displacement *relative* to the substrate which is moving with a constant velocity. (Before I was doing it wrong and I calculated the displacement relative to the loading plate.)

2 Results

In Carlson and Langer's paper, they divided events into three types: microscopic events, localized events, and delocalized events, from the small to large. They defined the "moment" M of an event to be

$$M = \sum_j \delta U_j, \tag{1}$$

where the sum is over all blocks which are displaced during the event, and δU_j is the relative displacement of the jth block to the substrate. At the same time, they also define the corresponding "magnitude" of the event is $\mu = \ln M$. According to their theoretical results, for the system with parameters α, $\iota = \xi/a$, ν, and $N = L/a$ (a is the equilibrium distance between nearest neighbors, unity by default), the smallest microscopic event is an event with only one moving block and its moment M_1 is close to $2\pi\nu/[\frac{1}{2\iota^2}]^[3/2]$; the moment of the largest localized event is $\tilde{M} = 2\xi/\alpha = 2\iota/\alpha$; and the moment of the largest delocalized events with all block moving could be $M_L = 2L = 2N$. By taking natural logarithm, we get the corresponding magnitude μ_1, $\tilde{\mu}$, and μ_L. For $N = 100$, $\nu = 0.01$, $\alpha = 2.5$, and $\iota = 10$, the moments M_1, $\tilde{M}$, and M_L) are 2.22×10^{-5}, 8.0, and 200.0, respectively. After taking the natural logarithm, the corresponding magnitudes (μ_1, $\tilde{\mu}$, and μ_L) are -10.72, 2.08, and 5.29.

In Carlson and Langer's paper, the magnitude distribution, rather than the moment distribution of events was calculated. In this way, the power-law distribution of moments becomes the exponential distribution of magnitudes. In order to campare our results to theirs, we also calculate the magnitude distribution. Figure 1 shows the magnitude distribution for $N = 100$, $\nu = 0.01$, $\alpha = 2.5$, and $\iota = 10$) with velocity cutoff $v_0 = 0.01$ and time step $\Delta t = 0.005$. The scaling range is roughly from -6.0 to 1.0 which is the same as Carlson and Langer's. The slope was fitted from -5.0 to 0.0 and is 0.95 ± 0.03 (indistinguishable from unity in Carlson and Langer's paper). Hence, we can say that this distribution agrees with Carlson and Langer.

Carefully choosing the time step and velocity cutoff may yield more accurate results. For example, if we let $v_0 = 0.001$ (ten times smaller), we get the slope 0.98 ± 0.02 shown in Figure 2; the scaling range is similar. The distribution for a larger cutoff value ($v_0 = 0.1$) is shown in Figure 3; the range of the linear regime is smaller and the slope is smaller. This is because the average speed for a microscopic event with s blocks moving at the same speed is $s \times 5.0 \times 10^{-5}$ according to the theory of Carlson and Langer ($s \times \nu/(2\iota^2)$ which is much less than ν for small s. If we use too large a velocity cutoff, we will shorten the real moving time of a block and neglect moving blocks with very small velocity. Of course, this depends how accurate the simulation can produce the trajectory which depends on the algorithm and the time step used.

I also changed the time step to see its effects on the magnitude distribution. Figure 4 shows that there only tiny differences for the larger events (localized and delocalized) when the time step is decreased from 0.01 to 0.001. However, the distributions of microscopic events shows bigger differences because the average speed of blocks in an microscopic event is much less than the load velocity $\nu = 0.01$. A large time step can not calculate small velocities accurately enough. So the distribution of microscopic events is much more sensitive to the algorithm and the time step than larger events.

Figure 5 shows two magnitude distributions for systems with the same parameters as above but at two different interaction ranges $R = 1$ and $R = 5$. For $R = 1$ there are obvious differences between the distribution of microscopic events and that of larger events: the distribution of microscopic events has its first peak at value μ_1 and the subsidiary peaks near μ_2, μ_3, etc. are visible, but are increasingly broadened because larger groups of blocks can slip with larger variations of their internal configurations. For larger events there are no such discrete peaks and the distribution can be described by a power law. For $R = 5$ such differences becomes unclear. So we expect for infinite range such difference will disappear and become one power law between μ_1 and $\tilde{\mu}$. Since there are not enough larger events whose magnitudes are close to $\tilde{\mu}$, which is the magnitude of the largest localized event predicted by Carlson and Langer, I fitted the slope from -10.0 to -2.0 and got 1.09 ± 0.03.

Figure 6 shows two magnitude distributions for two systems with $N =$ 100, and 500 respectively. The shapes of this two distributions show little diferences except that the peak of delocalized events shirts to right. This is due to the cutoff value of the magnitude for a delocalized event is $\mu_L = 2N$ which linearly depends the system size N. We should also note that a bigger system $N = 500$ decreases the slope of scaling range from 1.00 ± 0.02 to 0.87 ± 0.02 if we fit the data from -5.0 to 1.0.

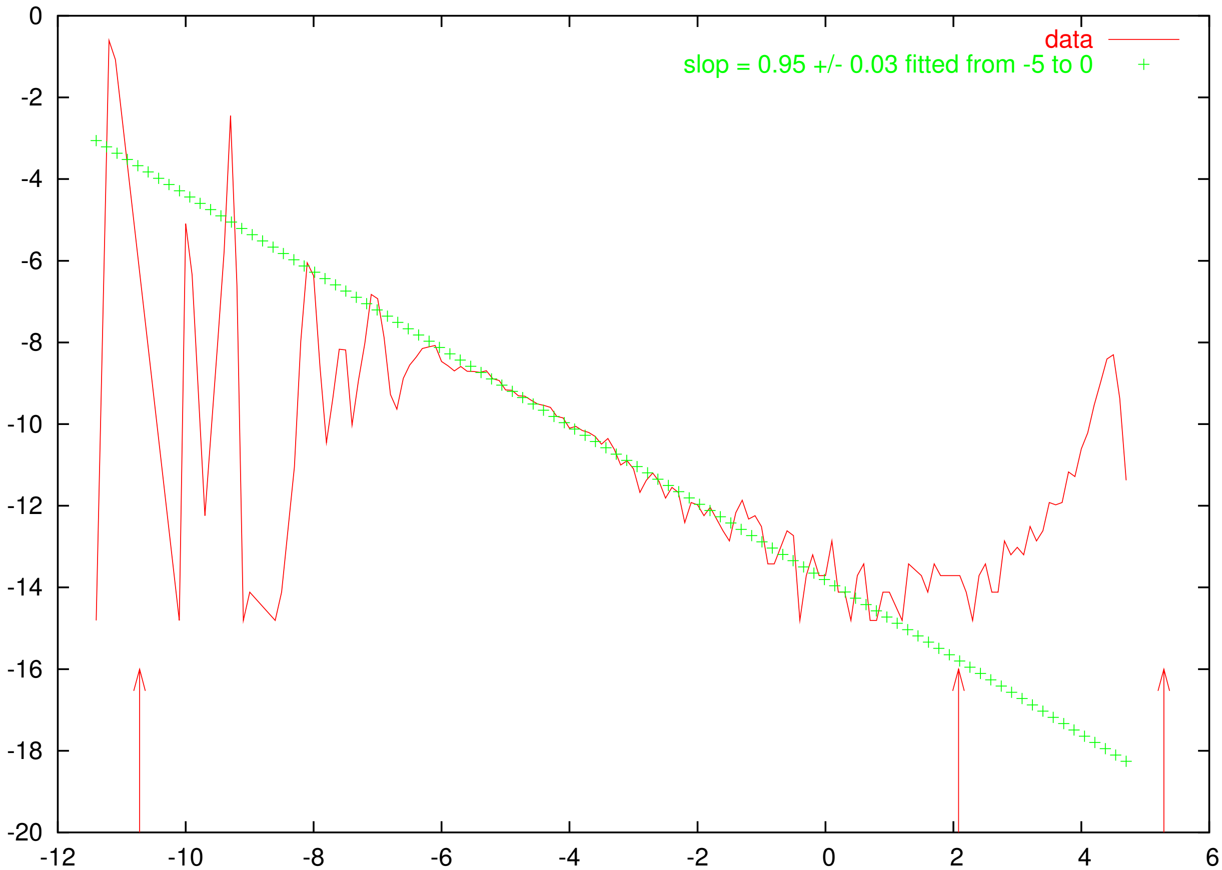

Figure 1: Plot of distribution magnitude (which is the log of moment) for $N = 100$, $\nu = 0.01$, $\alpha = 2.5$, $\iota = 10$, $R = 1$ with a velocity cutoff $v_0 = 0.01$ for $200,000\tau$. Note that this averaging time is the same as Carlson and Langer. The arrows show the positions of μ_1, $\tilde{\mu}$, and μ_L. The slope is fitted from -5.0 to 0.0 and is 0.95 ± 0.03. (Carlson and Langer claim that the slope is indistinguishable from unity.

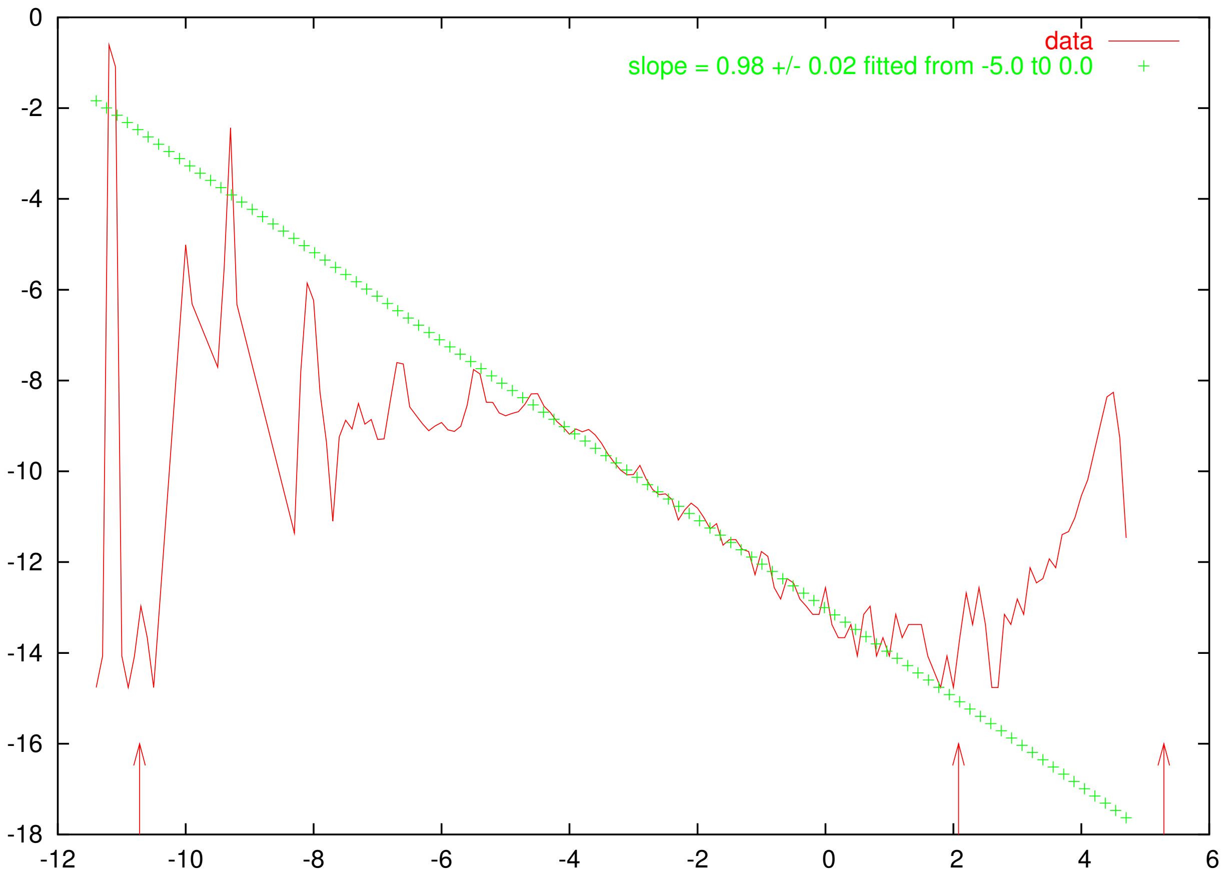

Figure 2: Same as Fig. 1, but with a smaller velocity cutoff $V_0 = 0.001$. The slope fitted from -5.0 to 0.0 is 0.98 ± 0.02 and is closer to unity.

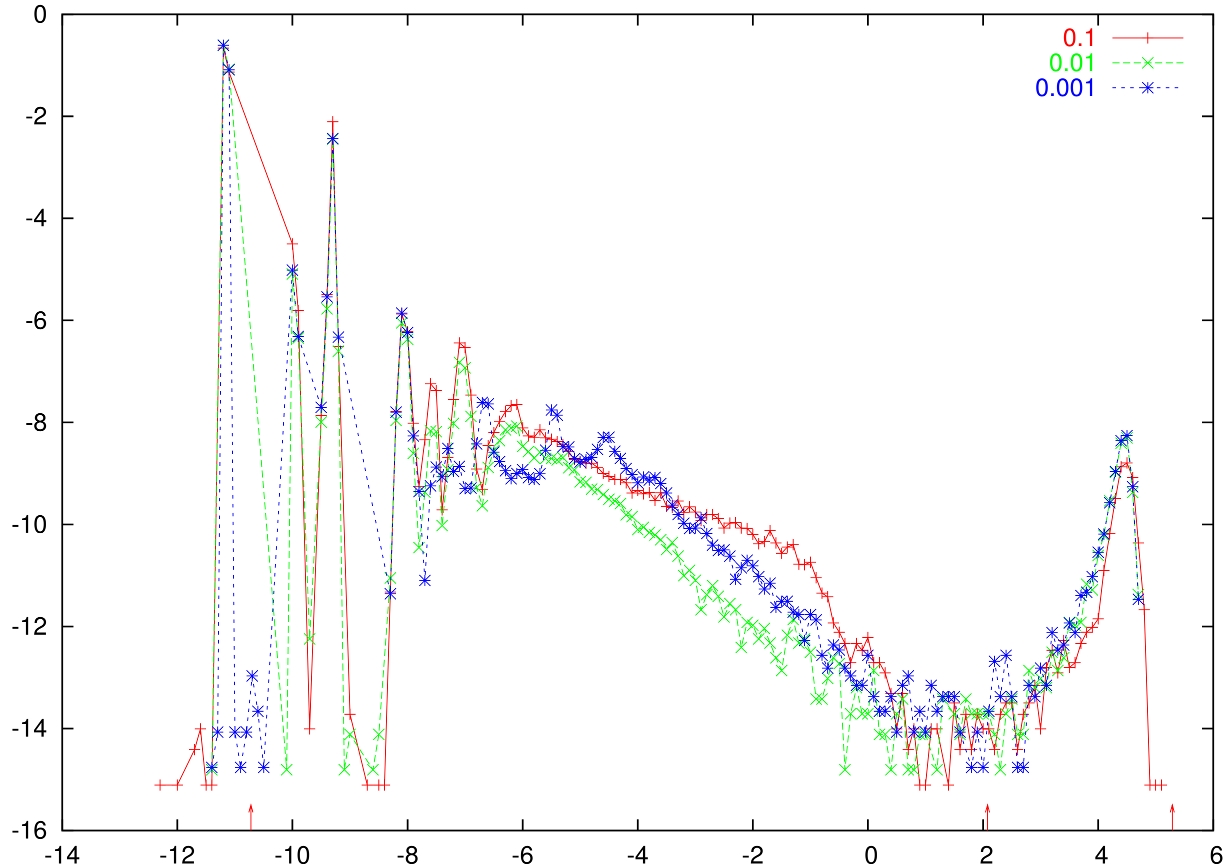

Figure 3: Same as Fig. 1, but with three different velocity cutoffs $V_0 = 0.1$, 0.01, and 0.001. The plot is as linear for the largest cut-off.

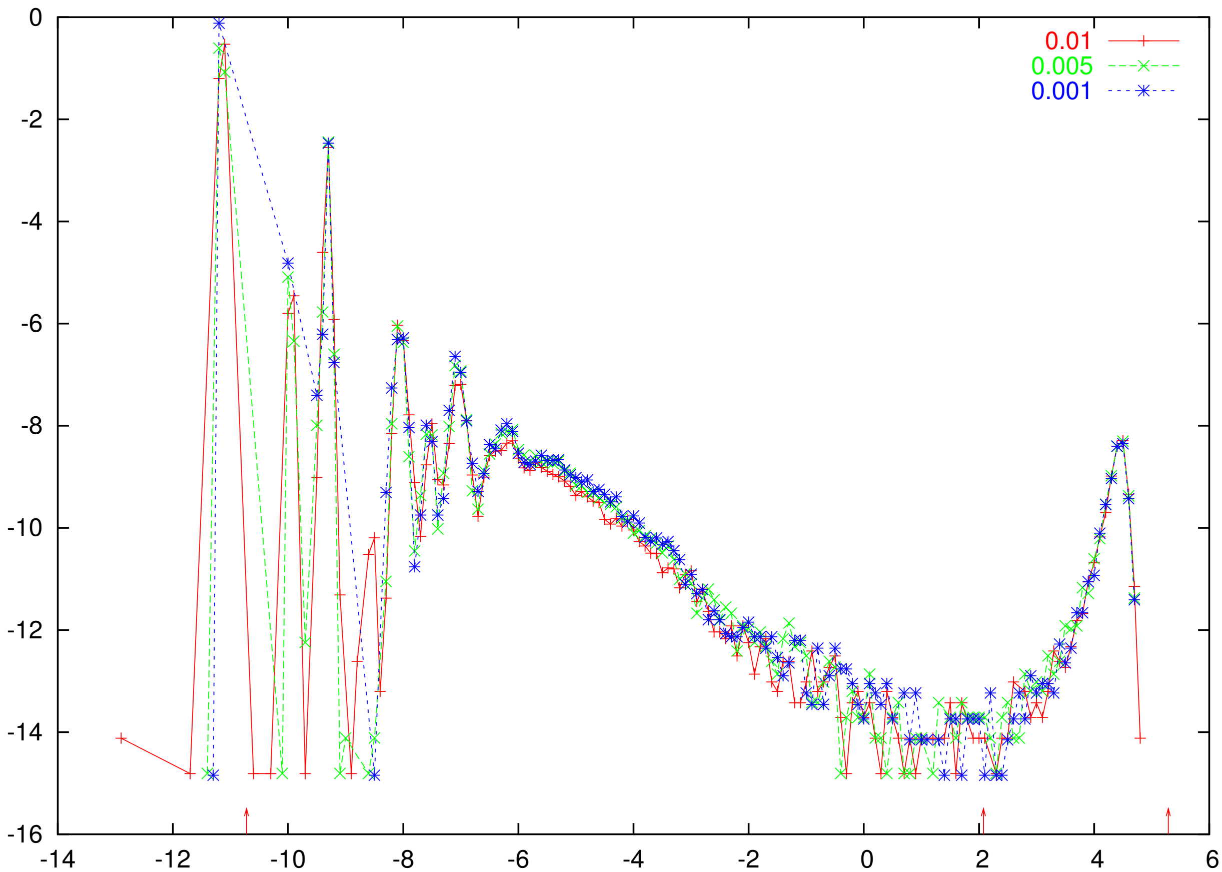

Figure 4: Same as Fig. 1, but for three different time steps $\Delta t = 0.01$, 0.005, and 0.001. The slope does not change much.There is almost no change for the peak at large magnitude.

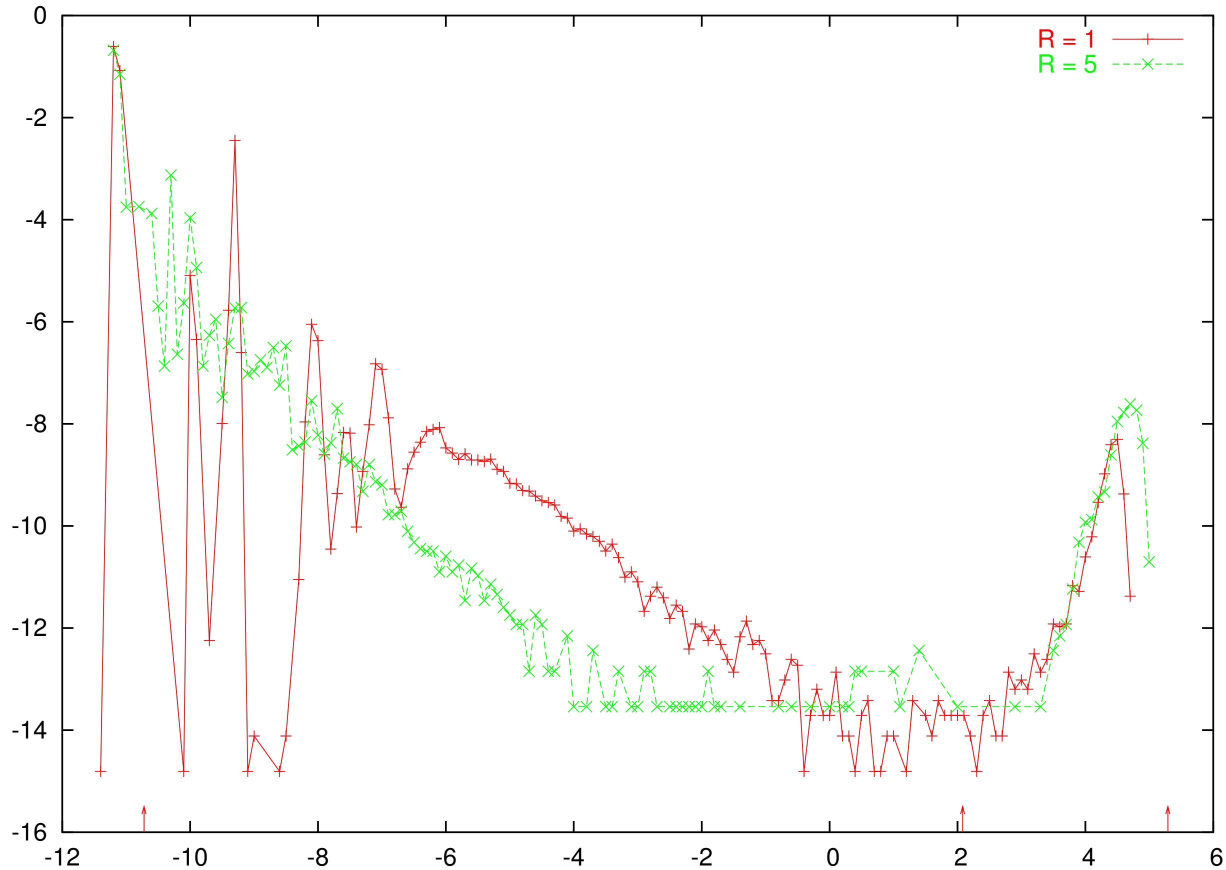

Figure 5: Same parameters as Fig. 1, but for two interaction ranges $R = 1$ and 5.

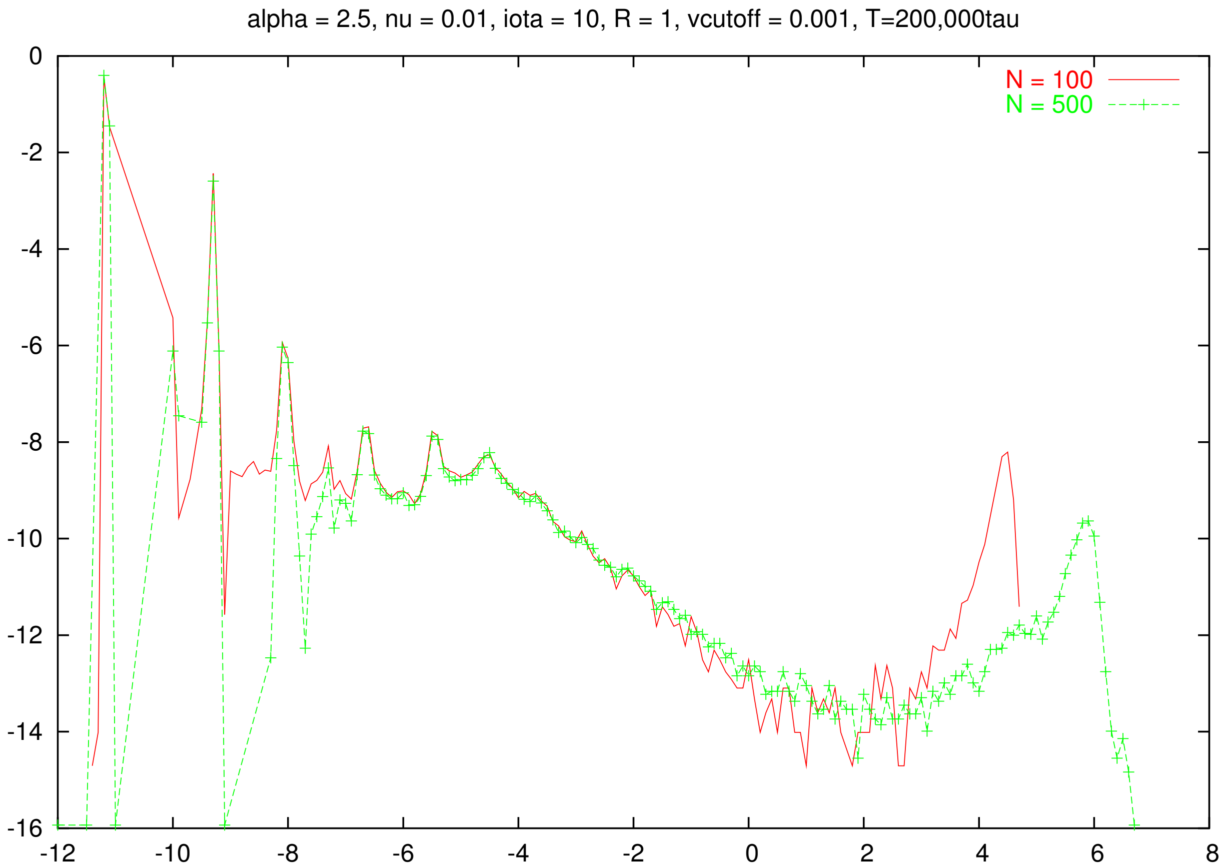

Figure 6: Same parameters as Fig. 2, but for two systems with $N = 100$ and 500.