Simulation of the Burridge-Knopoff Model with Long-Range Interactions

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Junchao Xia
Clark University
jcxia@physics.clarku.edu

in collaboration with
Harvey Gould

Clark University
hgould@physics.clarku.edu

William Klein

Boston University and Los Alamos National Laboratory
klein@buphy.bu.edu

John Rundle

University of California, Davis
jbrundle@ucdavis.edu

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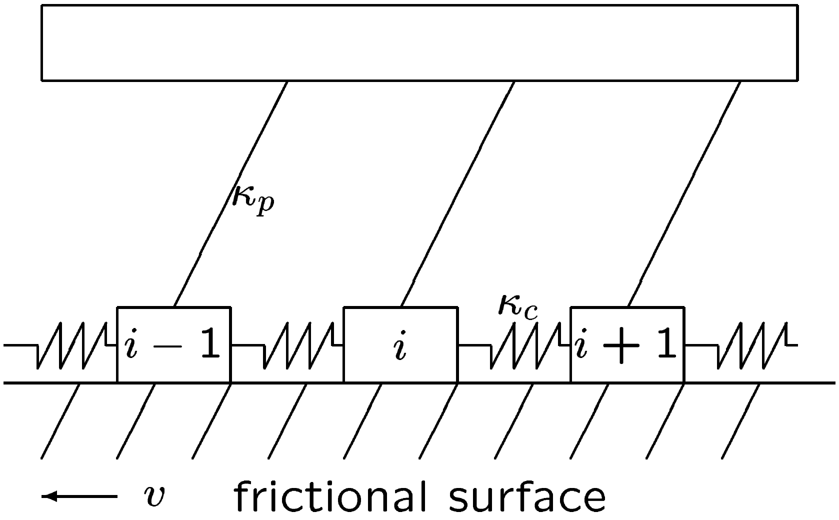
Summary

We simulate the Burridge-Knopoff model in one dimension for various interaction ranges.

1. The magnitude distribution of events (earthquakes) for nearest-neighbor interactions is similar to Carlson and Langer. One scaling region corresponding to localized events is found.

2. Near-mean-field effects are simulated by considering longer range interactions. Our results suggest the existence of two scaling regions, corresponding to different types of events. However, the nature of the events is not understood.

Burridge-Knopoff Model



Blocks of mass m are coupled by springs of strength κ_c and attached to a fixed surface by springs of strength κ_p. The blocks are in contact with the rough substrate which moves at speed v to the left:

$$m\ddot{x}_j = \kappa_c(x_{j+1} - 2x_j + x_{j-1}) - \kappa_p x_j - F(v + \dot{x}_j) \qquad (1)$$

x_j is the displacement of block j from equilibrium. The velocity-dependent frictional force F between blocks and surface has the form (Carlson and Langer, 1989)

$$F(\dot{x}) = F_0\phi(\dot{x}/\tilde{v}), \qquad \phi(y) = \frac{1}{1+|y|}\mathrm{sgn}(y)\,. \qquad (2)$$

A more realistic frictional form is (Carlson and Langer, 1991)

$$\phi(y) = \begin{cases} (1-\sigma)/(1+y/(1-\sigma)) & (y>0) \\ (-\infty, 1] & (y=0) \end{cases} \qquad (3)$$

Dimensionless Parameters

$$\tau = \omega_p t, \quad \omega_p^2 = \kappa_p/m, \quad x_j = (F_0/\kappa_p)u_j = D_0 u_j,$$

Equation of motion:

$$\ddot{u}_j = \ell^2(u_{j+1} - 2u_j + u_{j-1}) - u_j - \phi(2\alpha\nu + 2\alpha\dot{u}_j),$$

$$\ell^2 = \kappa_c/\kappa_p, \quad \nu = v/(\omega_p D_0), \quad 2\alpha = \omega_p D_0/\tilde{v}$$

(dot denotes differentiation with respect to τ.)

Default parameters: $\ell = 10$, $\nu = 0.01$, and $\alpha = 2.5$, as in Carlson and Langer (1989).

Free boundary conditions and random initial positions are used. In the zero-velocity limit, $\nu = 0.0$ and Eq. (2) for $\phi(y)$ is used with $\sigma = 0.01$.

Near-mean field systems are simulated by assuming $2R$ springs with coupling constant κ_c/R attached to each spring. The mean-field limit is equivalent to $R \to \infty$.

Scaling of Events

Definition of an event: At a given time, all moving blocks are identified. Two moving blocks ($v > v_0$) that are within the interaction range R are in the same event. An event is followed until none of the blocks are moving ($v < v_0$). The moment M of an event is

$$M = \sum_j \delta u_j. \tag{4}$$

The sum is over all blocks in the event; δu_j is the relative displacement of the jth block to the substrate. The corresponding magnitude of the event is $\mu = \ln M$. We are interested in the distribution of M and μ, $P(M)$ and $P(\mu)$.

Another distribution of interest is the number of blocks in an event.

Results

1. We obtain results similar to Carlson and Langer for the magnitude distribution (Fig. 1). $N = 1000$ gives results similar to $N = 100$.

2. For the more realistic friction force, Eq. (3) and the zero-loading velocity limit as Carlson and Langer (1991), we calculate the magnitude distribution for $N = 1000$ for $R = 1$ and 100 (Fig. 2). There is only one scaling range for $R = 1$ with exponent $b = 0.8$. (The slope appears to depend weakly on the loading velocity.) For $R = 100$ another scaling range with $b = 1.6$ appears for small μ.

3. The distribution of the number of blocks in an event for $N = 1000$ at $R = 1$, and 100 is shown in Fig. 3). The scaling region is better defined as R increases. We find $b \approx 2$ for $R = 100$. We also find that this region mainly arises from the fundamental events within the interaction range (Fig. 4).

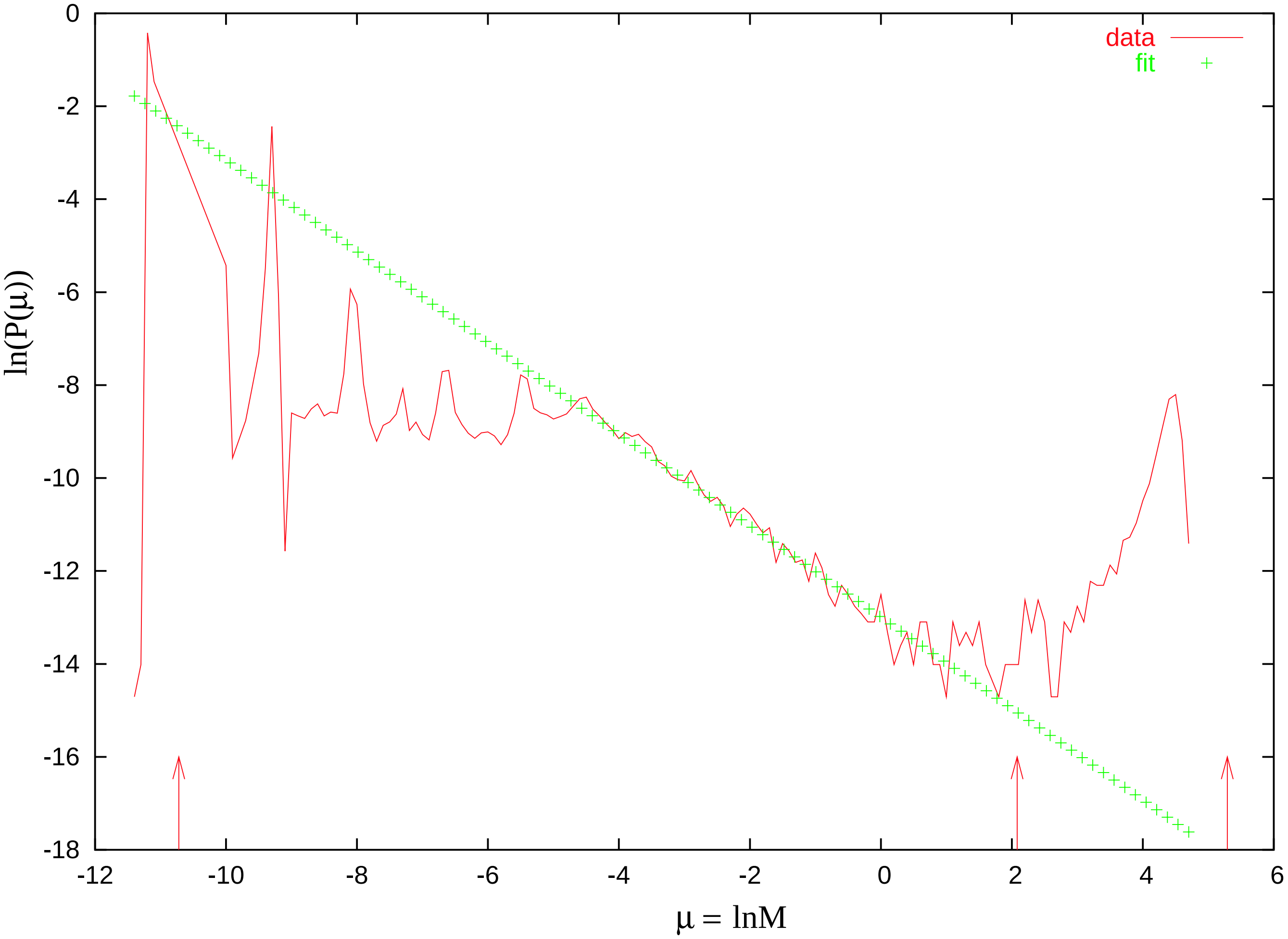


Figure 1. Magnitude distribution for $N = 100$, $R = 1$, and $v_0 = 0.001$. The arrows show the positions of μ_1, $\tilde{\mu}$, and μ_L. The scaling range is roughly from -6.0 to 1.0 which is the same as Carlson and Langer (1989). The slope fitted from -5.0 to 0 is $b = 0.98 \pm 0.02$.

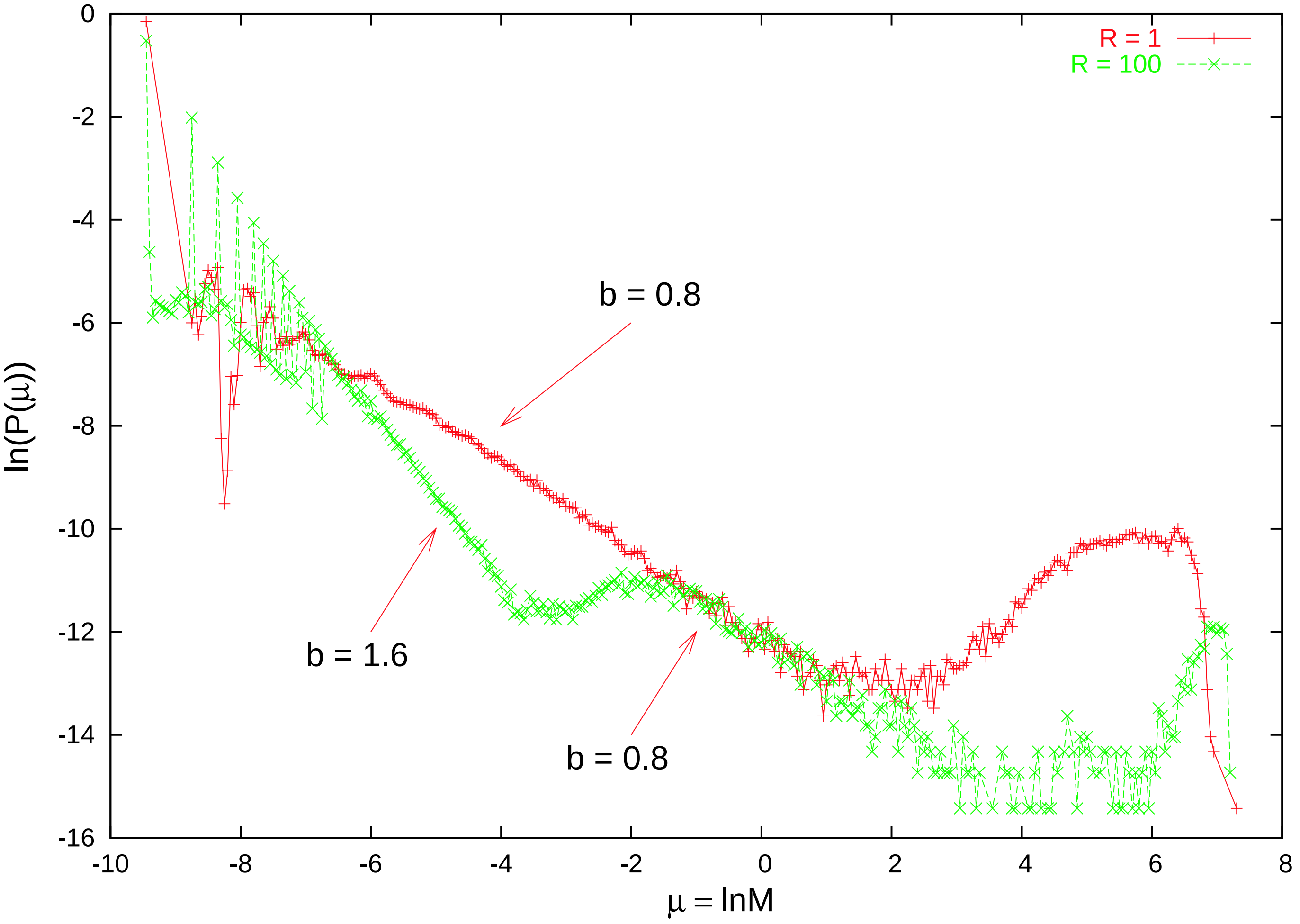


Figure 2. Magnitude distribution for $R = 1$ and 100 with zero loading velocity for $N = 1000$. There is only one scaling region for $R = 1$ with $b \approx 0.8$ similar to Carlson and Langer (1991). We find another scaling region with $b \approx 1.6$ for small μ for $R = 100$.

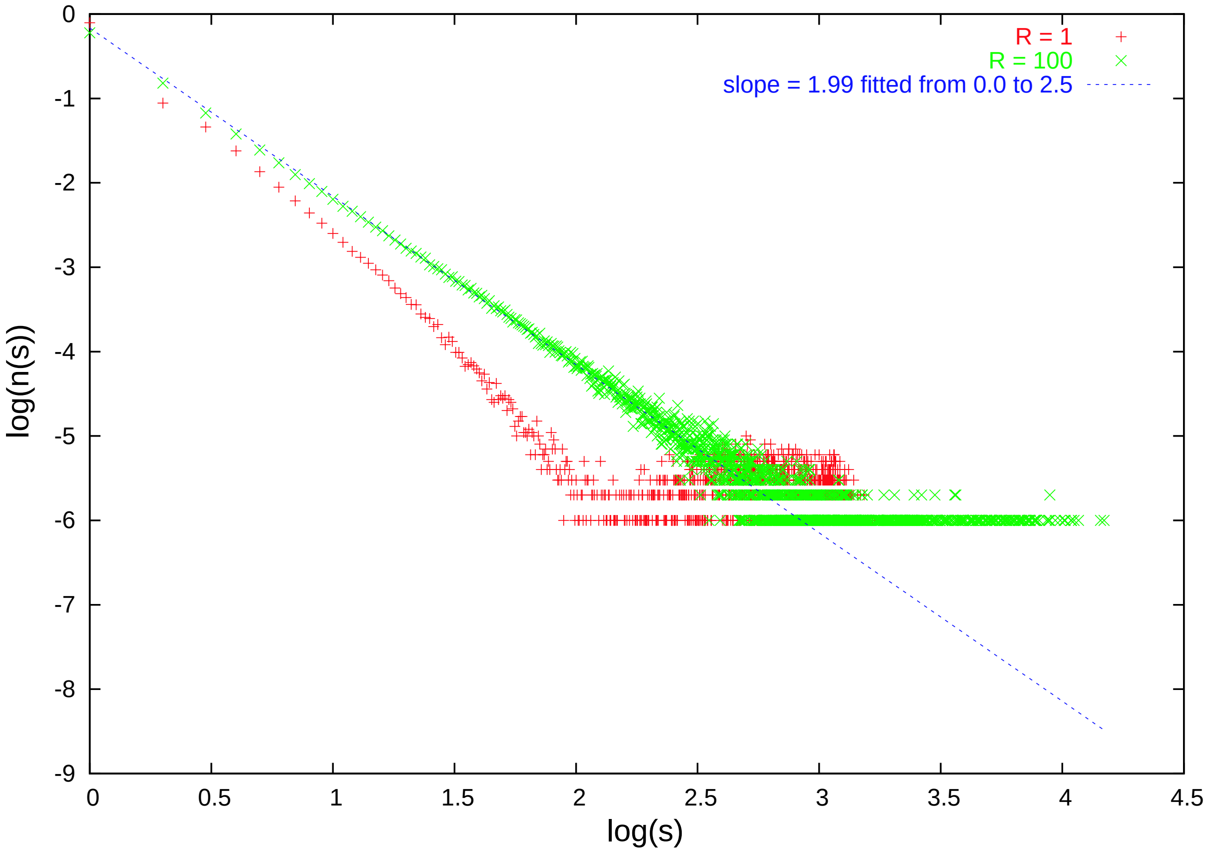


Figure 3. Distribution of the number of blocks (s) in an event for $N = 1000$ with zero loading velocity. For $R = 1$, there is no well-defined scaling range. For $R = 100$, there is apparent scaling with $b \approx 2$.

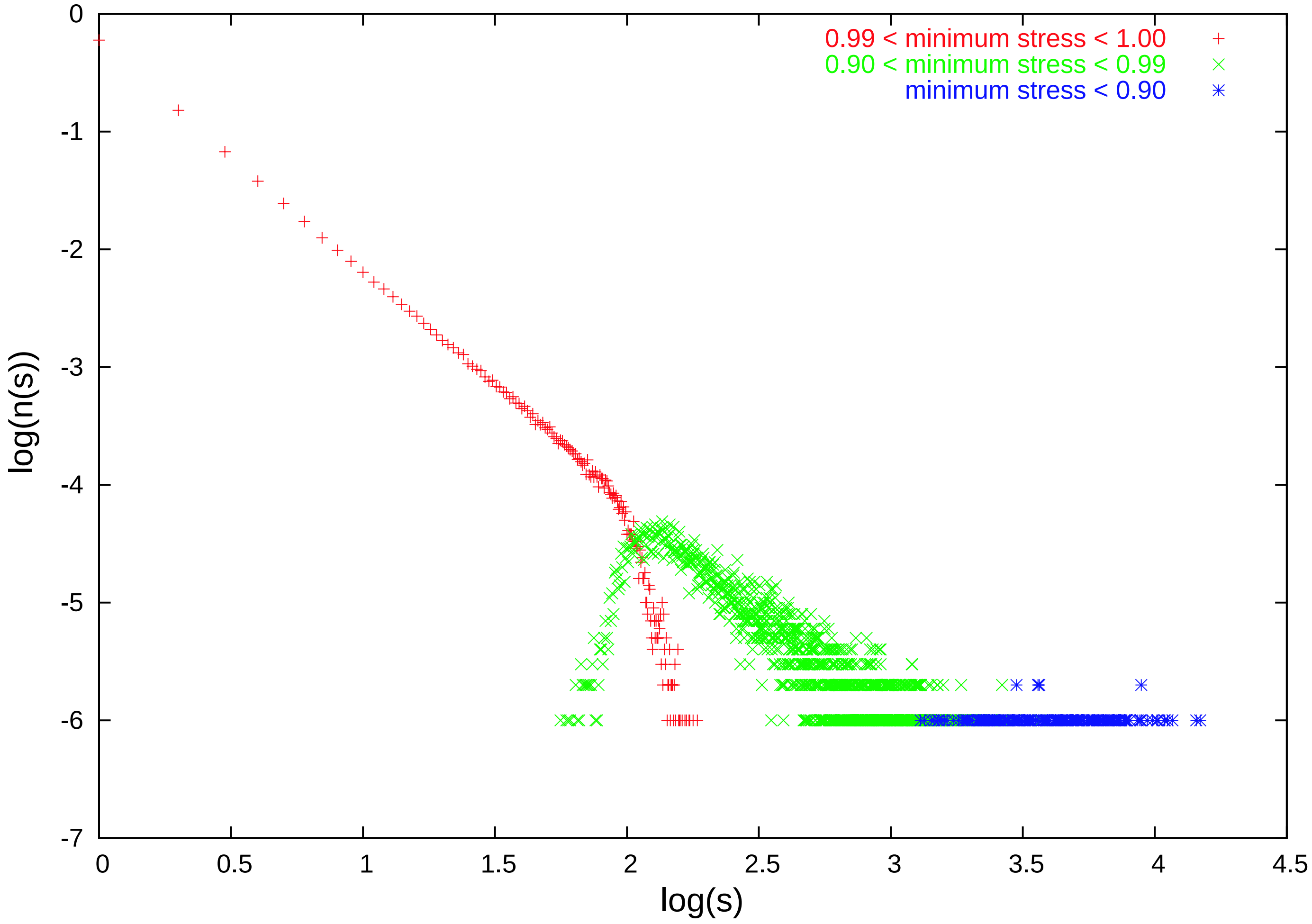


Figure 4. Distribution of the number of blocks (s) in an event for $R = 100$ in Fig. 3 is decomposed into three classes according to the stress on the blocks that make up an event just before the event occurs. This decomposition is similar to Anghel et al. for the RJB cellular automata model of earthquakes. I find that the slope is ≈ 2 for what is termed fundamental clusters and frustrated nucleation clusters.

Future Work

1. Consider larger N and R.
2. Determine the effects of the choice of the velocity cutoff v_0 on the distribution function. Preliminary results indicate that the slope of the scaling regions might be sensitive to the choice of v_0.
3. Consider model in two dimensions (similar results expected in the mean-field limit).

References

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