Simulation of Earthquakes Using the Burridge-Knopoff Model

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Summary

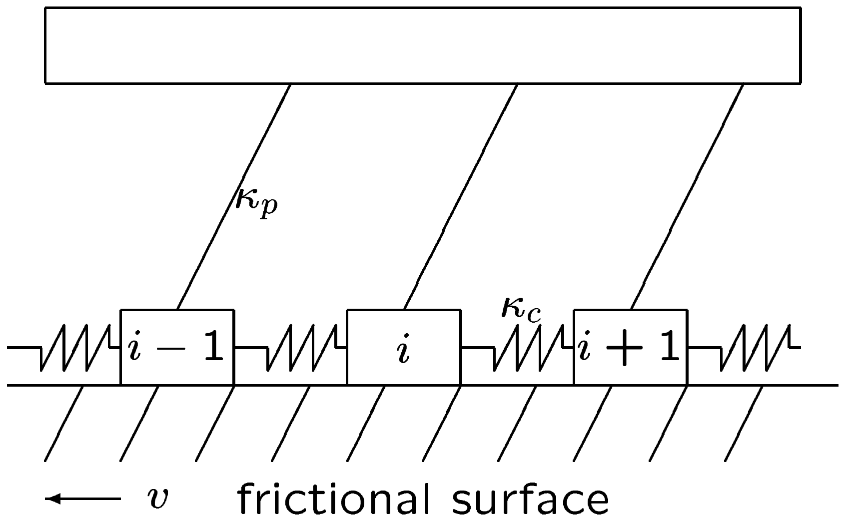
We simulate the Burridge-Knopoff model in one dimension for various interaction ranges. Our motivation is to determine if this dynamical model shows scaling behavior similar to various cellular automaton earthquake models.

1. The magnitude distribution of events (earthquakes) for nearest-neighbor interactions is similar to Carlson and Langer. Only one scaling region corresponding to localized events is found.

2. Near-mean-field effects are simulated by considering longer range interactions. Our results suggest the existence of two scaling regions, corresponding to different types of events.

3. Preliminary results suggest that as the interaction range increases, the scaling region corresponding to microscopic events becomes better defined, while the scaling region corresponding to localized events becomes poorly defined.

Burridge-Knopoff Model



Blocks of mass m are coupled by springs of strength κ_c and attached to fixed surface by springs of strength κ_p. Blocks in contact with rough substrate which moves at speed v to left:

$$m\ddot{x}_j = \kappa_c(x_{j+1} - 2x_j + x_{j-1}) - \kappa_p x_j - F(v + \dot{x}_j)$$

x_j is displacement of block j from equilibrium. The velocity-dependent frictional force F between blocks and surface has the form

$$\begin{aligned} F(\dot{x}) &= F_0 \phi(\dot{x}/\tilde{v}), \\ \phi(y) &= \frac{1}{1+|y|}\mathrm{sgn}(y)\,. \end{aligned}$$

$\tilde{v}$ is characteristic speed that characterizes velocity dependence of F. When a block is stuck, F can be any value between $\pm F_0$ in order to cancel the driving force from the springs.

Dimensionless Parameters

$$\tau = \omega_p t, \quad \omega_p^2 = \kappa_p/m, \quad x_j = (F_0/\kappa_p)u_j = D_0 u_j,$$

Equation of motion

$$\ddot{u}_j = \ell^2(u_{j+1} - 2u_j + u_{j-1}) - u_j - \phi(2\alpha\nu + 2\alpha\dot{u}_j),$$

$$\ell^2 = \kappa_c/\kappa_p, \quad \nu = v/(\omega_p D_0), \quad 2\alpha = \omega_p D_0/\tilde{v}$$

(dot denotes differentiation with respect to τ.)

The default parameters are $\ell = 10$, $\nu = 0.01$, and $\alpha = 2.5$, which are the same as in Carlson and Langer (PRA 1989). Free boundary conditions and random initial positions are used. All results shown are averaged over 200 000 τ after running for 10 000 τ to reach steady state.

Numerical Algorithm

Modification of RK4 algorithm due to Shampine and Watts (SWRK4):

$$
\begin{aligned}
k_{1v} &= a(x_n, v_n, t_n)\Delta t && \text{(same)}\\
k_{1x} &= v_n \Delta t && \text{(same)}\\
k_{2v} &= a(x_n + \frac{k_{1x}}{2}, v_n + \frac{k_{1v}}{2}, t_n + \frac{\Delta t}{2})\Delta t && \text{(same)}\\
k_{2x} &= (v_n + \frac{k_{1v}}{2})\Delta t && \text{(same)}\\
k_{3v} &= a(x_n + \frac{k_{1x}}{4} + \frac{k_{2x}}{4}, v_n + \frac{k_{1v}}{4} + \frac{k_{2v}}{4}, t_n + \frac{\Delta t}{2})\Delta t\\
k_{3x} &= (v_n + \frac{k_{1v}}{4} + \frac{k_{2v}}{4})\Delta t\\
k_{4v} &= a(x_n - k_{2x} + 2k_{3x}, v_n - k_{2v} + 2k_{3v}, t_n + \Delta t)\Delta t\\
k_{4x} &= (v_n - k_{2v} + 2k_{3v})\Delta t
\end{aligned}
$$

$$
\begin{aligned}
v_{n+1} &= v_n + \frac{1}{6}(k_{1v} + 4k_{3v} + k_{4v})\\
x_{n+1} &= x_n + \frac{1}{6}(k_{1x} + 4k_{3x} + k_{4x}).
\end{aligned}
$$

Nature of Blocks

A block that is "moving" at $t = t_{n-1}$ is **stuck** at time $t = t_n$ if

1. its speed, V_n, relative to the substrate is less than an arbitrary value, such as $V_0 = 0.001$;

2. the direction of its relative velocity is opposite to the total force (including friction) on it at $t = t_{n-1}$;

3. The driving force F_{drive} due to its neighbors and the loading plate only (neglecting friction) is less than F_0, the maximum value of the static friction force.

A stuck block will accelerate (slip) when the driving force is larger than F_0.

Calculation of total force:

If $F_{\text{drive}} < F_0$ and the block is stuck, then we choose the static friction force, f_s, so that it exactly cancels F_{drive} and the total force is zero. Otherwise, the maximum static friction force does not cancel F_{drive}, and we have to calculate the dynamic friction force f_d and add it to the driving force to get the total force on the block.

Definition of Event, Moment, and Magnitude

A moving block will be counted in an event if, (1) it has slipped at the beginning of the event, or (2) it begins to move after the birth of this event, but it is a neighbor of a block that is already part of this event.

An event can expand and shrink with time and die when all moving blocks in the event stop slipping.

The "moment" M of an event is

$$M = \sum_j \delta u_j, \tag{1}$$

where the sum is over all blocks that are displaced during the event, and δu_j is the relative displacement of the jth block to the substrate. The corresponding magnitude of the event is $\mu = \ln M$.

Let $\mathcal{H}(\mu)d\mu$ be the frequency of events, whose magnitudes are between μ and $\mu + d\mu$. According to Gutenberg and Richter, this function has the form

$$\mathcal{H}(\mu) \cong \frac{A}{M^b} = Ae^{-b\mu} \tag{2}$$

where A is a constant (independent of M) and $b \cong 1$.

Results

1. For $R = 1$, scaling region is only well defined for localized events (Fig. 1). Increasing the system size from $N = 100$ to 1000 have little effect on the scaling region (Fig. 2).

2. Longer-range interactions simulated by assuming $2R$ springs with coupling constant κ_c/R are attached to each spring. Mean-field limit equivalent to $R \to \infty$.

3. As R increases, a scaling region appears for the microscopic events and becomes better defined, while the scaling region corresponding to localized events is poorly defined (Fig. 3).

4. The results for $N = 1000$ show these scaling trends much clearer (Fig. 4).

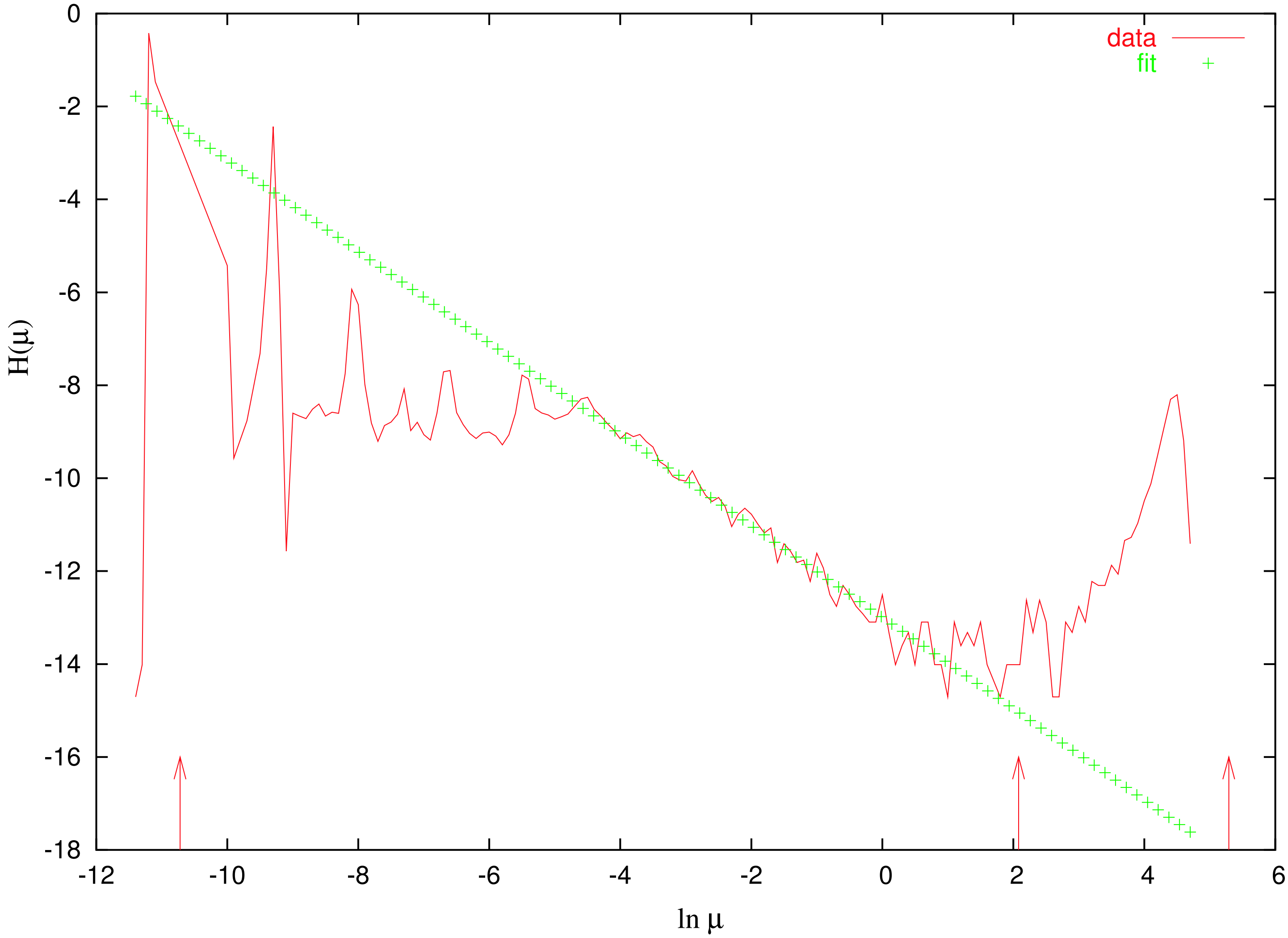


Figure 1. Magnitude distribution for $N = 100$, $R = 1$, and $v_0 = 0.001$. The arrows show the positions of μ_1, $\tilde{\mu}$, and μ_L. The scaling range is roughly from -6.0 to 1.0 which is the same as Carlson and Langer. The slope fitted from -5.0 to 0.0 is $b = 0.98 \pm 0.02$.

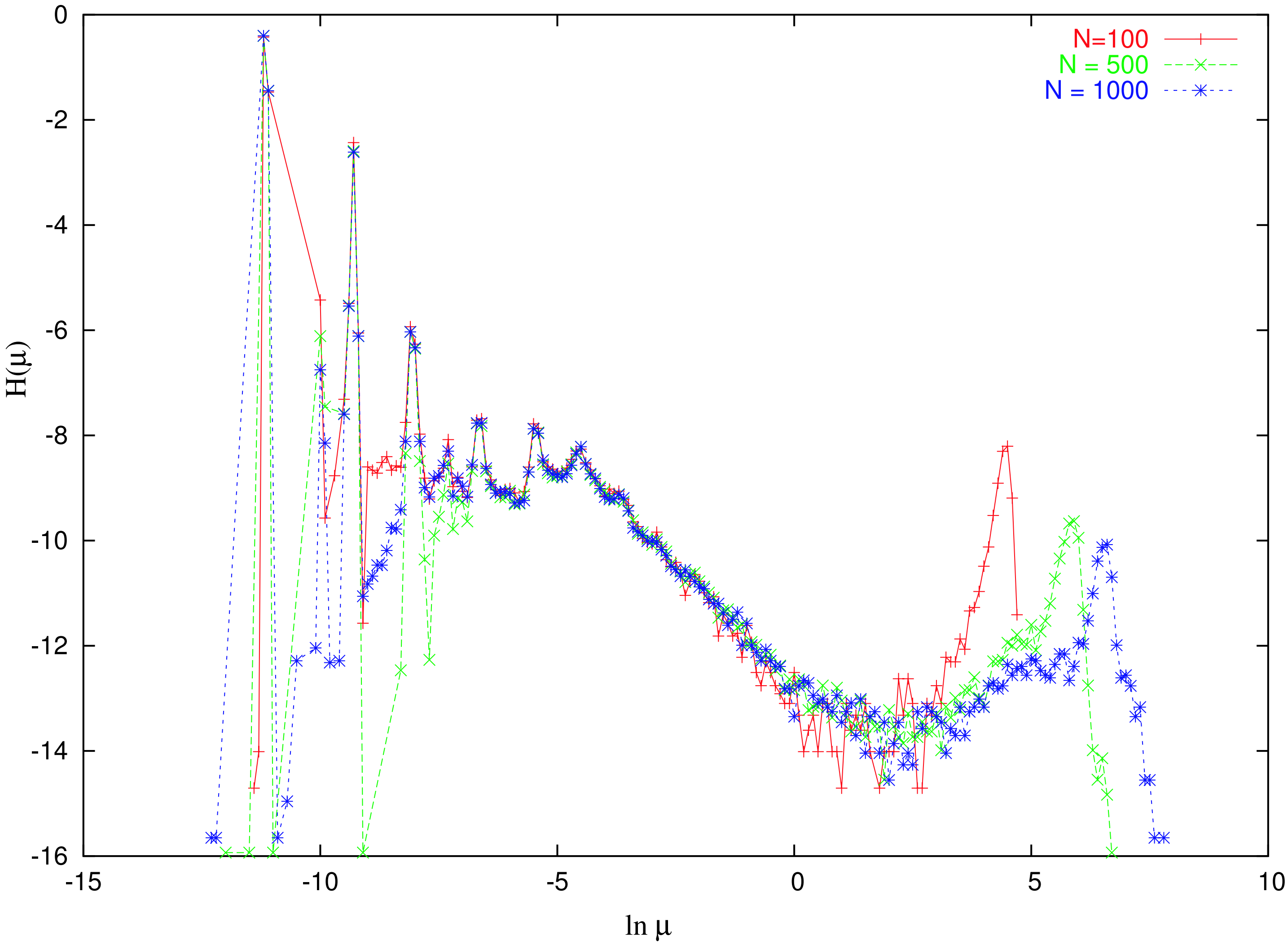


Figure 2. Magnitude distribution for $R = 1$ and $N = 100$, 500 and 1000. Note the absence of finite size effects.

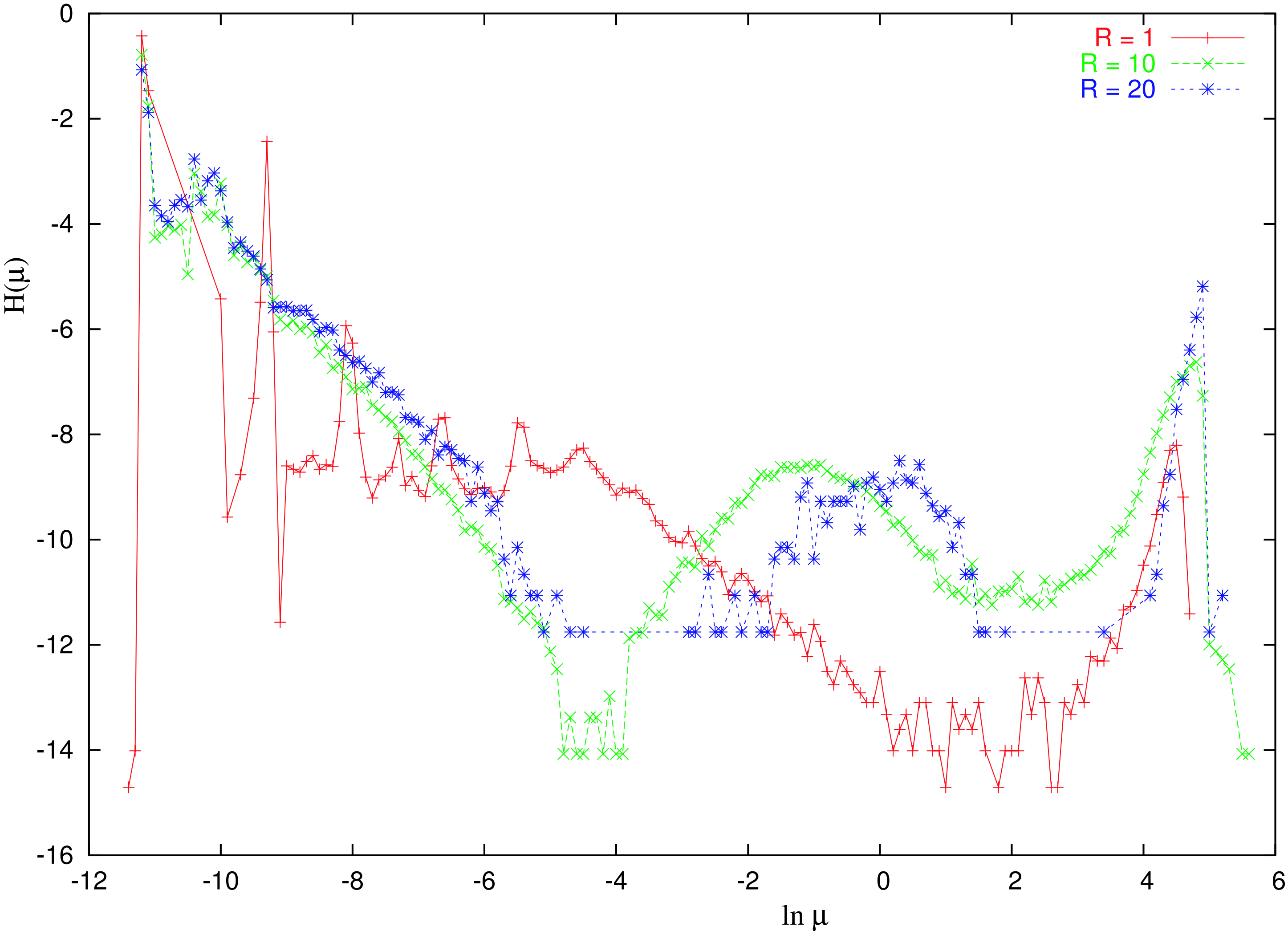


Figure 3. Magnitude distribution for $N = 100$ and $R = 1$, 10 and 20. Results are suggestive only because of finite size effect for $R > 1$.

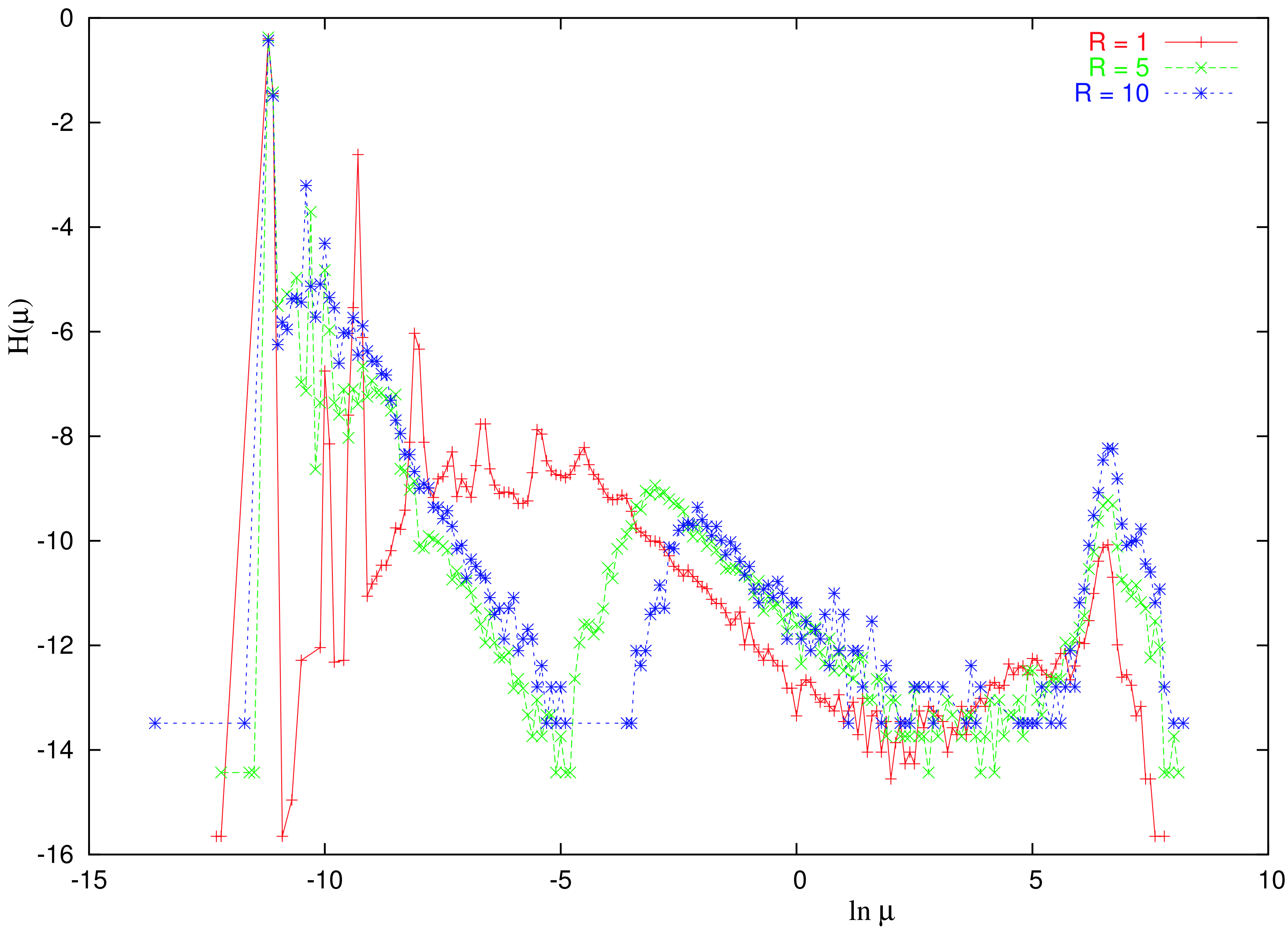


Figure 4. Magnitude distribution for $N = 1000$ and $R = 1$, 5, and 10. $R = 1$, only one scaling range: $\mu = -5.0$ to 2.0, $b = 0.96 \pm 0.02$. $R = 5$, two apparent scaling ranges: $\mu = -10.0$ to -5.0, $b_1 = 1.65 \pm 0.05$; $\mu = -3.0$ to 2.0, $b_2 = 0.90 \pm 0.03$. $R = 10$, two apparent scaling ranges: $\mu = -10.0$ to -5.0, $b_1 = 1.65 \pm 0.04$; $\mu = -2.0$ to 2.0, $b_2 = 0.91 \pm 0.09$.

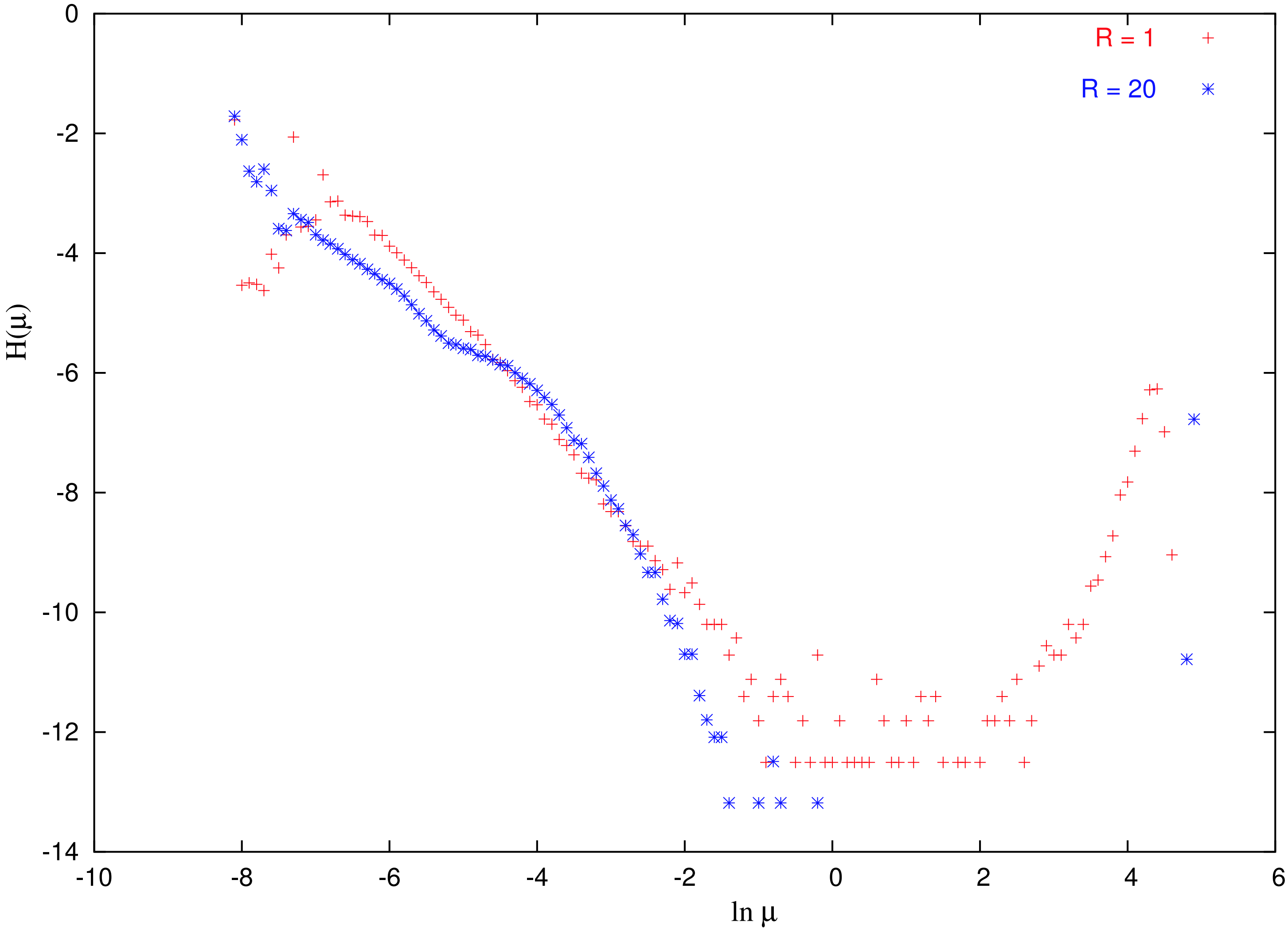


Figure 5. Magnitude distribution for $N = 100$ and $R = 1$ and 20. The way of loading the system is modified: Once one of the blocks slips, I stop loading until all blocks become stuck, instead of continuous loading as before. In comparison to Fig. 3, the results are much smoother. There is also more than one scaling range for $R > 1$. Because of finite size effects, the curves are only suggestive.

Future Work

1. Consider larger N and R and modify program so that it runs in parallel.

2. Consider model in two dimensions (similar results expected in mean-field limit).

References

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