We would like to demonstrate two aspects of Faraday’s Law in this lab. First that the voltage induced in a coil by a changing magnetic field is proportional to the numbers of turns in the coil, and second that the induced voltage is proportional to the rate of change of the magnetic field that passes through the coil. In addition you will be introduced to the oscilloscope used as a computer in a laboratory setting.

**Theory**

When we move a magnet through a coil of wire, we change the amount of magnetic flux in the coil and produce a current in the wire. The electromotive force (EMF) or voltage induced in the coil is given by Faraday's Law:

\[ \mathcal{E} = -N \frac{d\mathcal{B}}{dt} \]

where \( \frac{d\mathcal{B}}{dt} \) is the time rate of change of the flux and \( N \) is the Number of turns in the coil.

The experiment consists of dropping a magnet through a coil of wire while measuring the voltage across the coil. As the magnet falls toward the coil, the magnetic flux will increase producing a voltage proportional to the rate of change of the magnetic flux. As the magnet leaves the coil, the flux will start to decrease and the voltage will change direction.

Let’s look first at the voltage produced at any instant. Using Faraday's law from above and the definition of magnetic flux

\[ \mathcal{B} = AB \cos \theta \]

we can write,

\[ \mathcal{E} = NAB \frac{d\mathcal{B}}{dt} \]

because the area \( A \) is constant and the we will only take into consideration the perpendicular component of the magnetic field so \( \cos \theta = 1 \). The change of the field with time, \( \frac{dB}{dt} \), can be separated into two parts. First, there is the change in the magnetic field as you move closer or further from the magnet, \( \frac{dB}{dx} \). Second, there is the change in position of the magnet with time, \( \frac{dx}{dt} \). Mathematically we write:

\[ \frac{dB}{dt} = \frac{dB}{dx} \frac{dx}{dt} \]

Of course \( \frac{dx}{dt} \) is just the velocity of the magnet, which we write as \( v \). Putting this all together and writing it as a proportion so we can drop the constants we have
The quantity $\frac{dB}{dx}$ is a geometric property of the magnet, which is a constant for any particular magnet. Similarly, $A$, the area of the coils will be constant for all the coils in this experiment. Thus if you use the same magnet for all your experiments,

$$\oint dB = \oint N A \frac{dx}{dx} v(t).$$

Or, to say it in words, the voltage produced by the loop is proportional to the number of turns of the coil and the velocity of the magnet.

**Total Flux**

The last calculation involves a bit of calculus so we will explain it graphically first. The idea is that regardless of how fast the magnet is moving, the total flux going through the magnet is the same. If we add up the change in flux times the time increment, as the magnet passes through the magnet, the total flux that passes through the magnet should be the same. This is a sum or better yet, an integral. Lets look at it graphically first.

The voltage we measure from the loop of wire is proportional to the change in magnetic flux in the loop divided by the change in time. This is Faraday’s law, the first equation in this lab write-up. Let’s look at an arbitrary graph of voltage vs. time, for a loop of wire with a magnet going through it.
This graph says that the magnetic field was becoming greater during the whole time. Even between 6 and 11 seconds and 22 and 30 seconds the field was increasing. Remember if the magnetic field had stopped changing the voltage would have been zero.

Lets look at the section between 22 and 30 seconds. During this time the voltage was 3.6 V. Our equation says:

\[ V = N \frac{\Delta B}{\Delta t} \]

or

\[ 3.6V = N \frac{\Delta B}{\Delta t} \]

So for this period of time the change in flux per unit time was constant or in other words the magnetic field was increasing at a constant rate. (Don’t worry about the negative sign, just flipping the magnetic poles will make it positive).

Now if we want to calculate the total magnetic flux that passed through the loop for the period of time between 22 and 30 seconds we simply multiply the voltage times the change in time, which is 8 seconds for this example (and divide by N if we want to compare different coils.) If we do this on the graph, we can see that this product is just the area under the curve.

This is exactly what an integral does for us, it sums up the area under a curve. Since the voltage is proportional to the change in flux, if we integrate the voltage, this should equal the total change in flux. Mathematically as before,

\[ \Phi = NA \frac{d\Phi}{dt} \]

If we use \( V(t) \) for \( \Phi \), which is the voltage we measure, and integrate both sides of the equation we have,

\[ \int_{t_1}^{t_2} V(t)dt = \int_{t_1}^{t_2} N \frac{d\Phi}{dt} \]

or

\[ \int_{t_1}^{t_2} V(t)dt = \int_{t_1}^{t_2} N \Phi \]

if we cancel out the time differential. The integral on the right is trivial so,

\[ \int_{t_1}^{t_2} V(t)dt = \int_{t_1}^{t_2} N \Phi = \int_{t_1}^{t_2} N(\Phi_{t_2} - \Phi_{t_1}) \]

that is the integral of the voltage over time is equal to the number of turns on the coil times the change in flux. Thus as before, the area under the voltage curve is equal to the change in flux through the loop of wire.
**Experimental**

For this experiment, you will use a digital oscilloscope to collect data at a very quick rate, to record the voltage as a function of time induced by a magnet falling through a loop of wire. The oscilloscope can record the voltage on the coil 1000 or more times at intervals of 1X10^-5 s (or less, you can adjust this if necessary) and display the results on the screen. It will also integrate the data for you and store the results if they are needed in the future.

From the voltage vs. time data you will be able to read the maximum induced voltage which is proportional to the velocity of the magnet and the number of turns on the coil. The velocity of the magnet can be calculated by knowing the height from which you drop the magnet, (remember the first semester?) the number of turns in the coil will be given.

From the integrated data, you will be able to verify that the total change in flux through the magnet is zero and that the maximum flux passing through the coil is constant (when the number of turns in the coil is taken into account).

**Procedure**

For the three different coils available, drop the magnet from at least four different heights (at least twelve trials). Record the maximum voltage, both positive and negative, the time at which the maximums occur, and the height of the integrated curve.

You will get a table to fill out in lab and a piece of graph paper. To finish this lab (i.e. to get credit for it) you need to:

- Fill in the data for the table. This includes the twelve trials described above.

- Make a graph of velocity vs. voltage peak height for each coil on the same piece of graph paper.

- Use all the wisdom you can muster during the experiment.

**Remember to always use the same magnet!!**